

# Detection Of Isomorphism Among Kinematic Chains Using Link-Joint-Loop Adjacency

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## Abstract

An important aspect in the field of mechanical engineering is to deal with the transmission of motion to meet the specific requirement. For fulfilling this need various mechanical elements such as linkages, levers, gear-trains, eccentrics, cams, etc. are commonly adopted. Hence, the structural analysis of kinematic chains and synthesis of linkages becomes a major area of study in mechanical engineering. At the conceptual stage of design, it is always required to select the best possible kinematic chain for the specified task. Hence, it becomes necessary to do the structural analysis and synthesis of all the possible distinct kinematic chains available with specified number of links and degrees of freedom. Basically, the results of structural synthesis ends with the generation of distinct kinematic chains but it does not lead to logical conclusion unless the capabilities of the structures are explored. How accurately a kinematic chain is generated with specified number of links and degree of freedom depends upon how accurately the chain is tested for isomorphism. Past researchers had developed some methods to test a kinematic chain for isomorphism but many of these methods fail at some stage or the other. Many of the methods that are reported to test for isomorphism and to identify distinct inversions in structural analysis are either based on link-link adjacency, link-joint adjacency or link-loop adjacency. But all the three important features of kinematic chains viz. link-joint-loop have not been considered for the exact structural assessment. Keeping this in view, quantitative methods are being developed which includes link-joint-loop adjacency to detect isomorphism among kinematic chains. The developed method for testing isomorphism also reveals other characteristics of kinematic chain and deliver results in many aspects e.g. efficiency, reliability, computational time, etc. Since, all the three features of kinematic chains viz. link-joint-loop are considered to analyze the structural aspect and its relation to function; the expected outcome will reveal some kinematic characteristics of the chain with high degree of reliability and efficiency for application in planar manipulators from the viewpoint of workspace.

**Index Terms** – Kinematic chains, Structural analysis, Synthesis of linkages, Structural synthesis, Isomorphism

## I. INTRODUCTION

In mechanical engineering design, the synthesis and analysis of kinematic chains is an important factor and hence a lot of researchers have made their efforts in studying the various aspects of mechanisms. In that context, detection of isomorphism among kinematic chains is very

essential for which time and effort is dedicated for evolving a technique which is trustworthy and proficient. In the early 70's, Uicker and Raicu [1] presented characteristic polynomial approach for detecting isomorphism of two kinematic chains. Later, Mruthyunjaya and Balasubramanian [2] proposed a degree matrix method while Ambekar and Agrawal [3] developed minimum code for

testing of isomorphism. Agrawal and Rao [4] attempted to develop computationally simple and efficient analytical methods using matrices. Rao and Raju [5] developed the hamming number technique while Hwang and Hwang [6] detected isomorphism by degree codes of contracted link adjacency matrices of kinematic chains. Rao and Rao [7] compared loop hamming values of links and chains for testing of isomorphism. Chu and Cao [8] originated link's adjacent-chain table to identify isomorphism. Shende and Rao [9] applied graph theory to convert kinematic chains into their equivalent graphs and compared for isomorphism. Tischler et. al. [10] presented a new orderly method for synthesizing kinematic chains. A computer aided method is proposed by Yadav et. al. [11, 12, 13] for detecting isomorphism using a new invariant called the arranged sequence of modified total distance ranks of all the links using the concept of modified distance, link-link multiplicity distance and link-path code. Quantitative methods based on link assortment, joint assortment and loop type and their adjacency is presented by Rao and Anne [14]. A new method based on an artificial neural network (ANN) technique is presented by Kong and Zhang [15] to identify the isomorphism. Rao [16] utilized fuzzy logic to investigate isomorphism. Rao and Pathapati [17] reported loop concept to reveal simultaneously chain is isomorphic, link is isomorphic, and type of freedom with no extra computational effort. Rao [18] presented genetic algorithm for testing isomorphism. Rao and Deshmukh [19] developed a method which obviates the test of isomorphism thus resulting in saving of time, space and effort to identify the distinct chains. Chang et. al. [20] developed a new method based on eigenvectors and eigenvalues. Mruthyunjaya presented a broad review of the extensive literature available on the subject kinematic structure with a view to trace its history highlighting major trends and discussing significant contributions. Xiao et. al. [22] presented two novel evolutionary approaches – ant algorithm (AA) and artificial immune system (AIS) to identify isomorphism. Srinath and Rao [23] used concept of correlation while Ding and Huang [24] presented perimeter topological graph to detect isomorphism. Bal, Deshmukh and Jagadeesh [25] defined link invariant functions based on distance matrix and

loops of a kinematic chain. Zeng et. al. [26] proposed a fast deterministic algorithm called the dividing and matching algorithm (DMA) and representing a kinematic chain uniquely by a graph to identify isomorphism. Rizvi et. al. [27] presented a new algorithm using adjacency matrices for determining the possible distinct inversions from a kinematic chain. Romaniak [28] reviewed the methods for identifying the isomorphism of kinematic chains suggested by researchers is contained in this study, including hamming number technique, eigenvalues and eigenvectors, perimeter graphs, dividing and matching vertices. Kamesh et. al. [29] proposed a novel and simple algorithm based on graph theory by which elimination of isomorphic chains can be done. Shukla et. al. [30] proposed gradient matrices, based on gradient analogy, to distinctly denote the structure of each kinematic chain which is applied successfully for detection of isomorphism and distinct inversions to all known families of kinematic chains i.e. single degree of freedom, multi degree of freedom, and detection of isomorphism in graphs.

Most of the methods in structural analysis have been reported to test isomorphism and inversions, but the methods reported so far is either based on link-link adjacency, link-joint adjacency or link-loop adjacency. But all the three important features of kinematic chains viz. link-joint-loop have not been considered for the exact assessment. Keeping in view, an attempt has been made in this paper to develop a method to detect isomorphism among kinematic chains which includes link-joint-loop adjacency thereby increasing the uniqueness.

## II. KINEMATIC CHAIN NOTATION AND REPRESENTATION

Consider a kinematic chain consisting of eight links (four ternary & four binary links) with single degree of freedom having ten revolute joints as shown in Fig. 1.



<b>2</b>	2	3	3	2	3	3	2	4	2	3	<b>27</b>
	0	10	11	0	0	0	0	9	0	0	<b>30</b>
<b>3</b>	3	2	3	3	2	3	3	2	3	2	<b>26</b>
	0	0	11	11	0	0	0	0	0	0	<b>22</b>
<b>4</b>	3	3	2	3	3	2	3	3	2	3	<b>27</b>
	0	0	0	11	11	0	0	0	0	10	<b>32</b>
<b>5</b>	2	3	3	2	3	3	2	3	3	2	<b>26</b>
	0	0	0	0	11	11	0	0	0	0	<b>22</b>
<b>6</b>	3	2	3	3	2	3	4	2	2	3	<b>27</b>
	10	0	0	0	0	11	9	0	0	0	<b>30</b>
<b>7</b>	2	2	2	3	3	2	4	4	3	2	<b>27</b>
	0	0	0	0	0	0	9	9	10	0	<b>28</b>
<b>8</b>	3	3	3	2	2	3	2	2	3	3	<b>26</b>
	0	0	0	0	0	0	0	0	10	10	<b>20</b>
<b>Link String Values</b>											
	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>LJL Value</b>		
	<b>27</b>	<b>27</b>	<b>26</b>	<b>27</b>	<b>26</b>	<b>27</b>	<b>27</b>	<b>26</b>	<b>213</b>		
	<b>20</b>	<b>30</b>	<b>22</b>	<b>32</b>	<b>22</b>	<b>30</b>	<b>28</b>	<b>20</b>	<b>204</b>		

Now the string value of a particular link in relation with the joint and loop values is calculated by summation of the numerators and the denominators separately for computation purpose only, as shown below:

String Value = Summation of joint values

Summation of loop values

i.e. for Link 1, String Value  
 $= 3+3+2+3+3+2+2+2+3+4 = 27$

$10+10+0+0+0+0+0+0 = 20$

Here the numbers 27 and 20 are not computed as fraction but individually shows respective joint and loop values of Link 1.

Now the link-joint-loop (LJL) value of a particular kinematic chain is calculated by the summation of all the joint values of all the links along with summation of all the loop values of all the links participating in the formation of loop, separately, as shown below:

LJL Value = Sum of all joint values of all the Links

Sum of all loop values participating in the formation of loops

i.e.  $LJLV = \frac{27+27+26+27+26+27+27+26}{20+30+22+32+22+30+28+20} = \frac{213}{204}$

Hence, for kinematic chain of Fig. 1, the LJL value comes out to be **213/204**. Alternately, a schematic presentation of LJL Value comprising joint and loop values is shown below:

**LJLV (Fig. 1) = [213/204] {2(20), 2(22), 3(26), 5(27), 1(28), 2(30), 1(32)}**

**IV. DETECTION OF ISOMORPHISM**

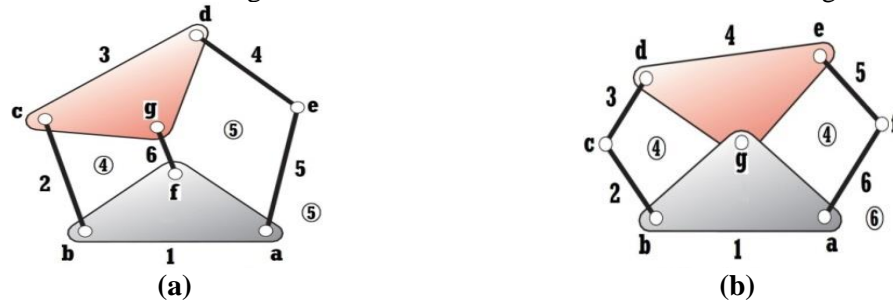
The above mentioned procedure is applied for all the available chains with fixed number of links and degree of freedom. It is observed that two isomorphic chains shows same LJL values as well as string values however distinct chains have different LJL values and if somehow their LJL

values are same, their string values are different. Application of the concept is illustrated with the help of several examples.

The method is applied to all the available kinematic chains as per literature up to 10-links and having one, two and three degrees of freedom. All the 16 eight bar 1-dof chains, 40 nine-bar 2-dof chains, 230 ten-bar 1-dof chains and 98 ten-bar 3-dof chains have been tested for isomorphism. All of them have yielded distinct LJL values or string values. The results in the

form schematic presentation of kinematic chains with 8-links 1-dof and with 9-links 2-dof is included in the appendix I and II. An example of chains with higher number of links cited from literature sources is also included in the present work to establish its reliability.

**Example 1:** It has been reported that there are two distinct chains available with 6 links and 1-dof i.e. Stephenson’s chain and Watt’s chain [3] shown in Fig. 2.



**Fig. 2: (a) Stephenson’s Chain and (b) Watt’s Chain**

Hence, their string values and LJL values are tabulated below in Table 2 which shows that these two chains have different LJL values and are non-isomorphic.

**Tab. 2: String Values & LJL Values of Stephenson & Watt Chain**

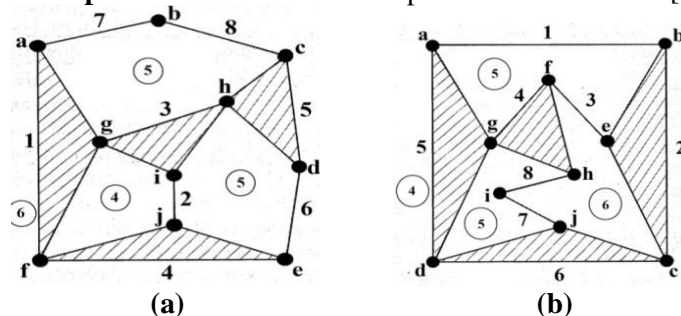
Chain	String Value						LJL Value
Stephenson Chain	18	17	18	17	17	17	<b>104</b>
	28	18	28	20	20	18	<b>132</b>
Watt Chain	18	17	17	18	17	17	<b>104</b>
	28	20	20	28	20	20	<b>136</b>

The schematic presentation of above chains will be:

$$LJLV_{sc} = [104/132] \{4(17), 2(18), 2(18), 2(20), 2(28)\}$$

$$LJLV_{wc} = [104/136] \{4(17), 2(18), 4(20), 2(28)\}$$

**Example 2:** Consider two isomorphic kinematic chains [21] having 8 links with 1-dof as shown in Fig. 3.



**Fig. 3: A pair of 8-links, 1-dof isomorphic chains [21]**

Now the string values and LJL values for pair of kinematic chains shown in Fig. 2 are tabulated in Tables 3 and 4.

**Tab. 3: String Values & LJL Values of chain in Fig. 3(a)**

1	2	3	4	5	6	7	8	LJL Value
27	27	27	27	27	26	26	27	<b>214</b>
30	18	28	30	32	22	22	22	<b>204</b>

**Tab. 4: String Values & LJL Values of chain in Fig. 3(b)**

1	2	3	4	5	6	7	8	LJL Value
27	27	26	27	27	27	26	27	<b>214</b>
18	30	22	32	28	30	22	22	<b>204</b>

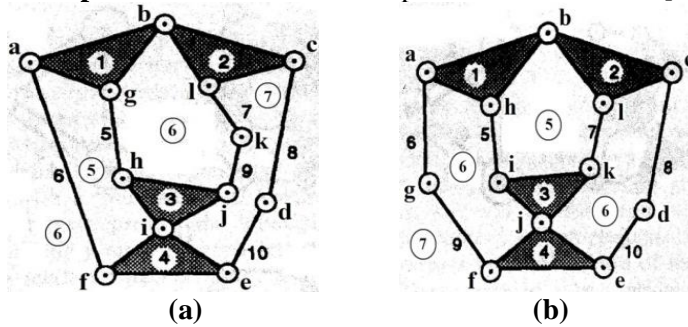
Alternately, schematic presentation will be:

$$LJLV_{3a} = [214/204] \{1(18), 3(22), 2(26), 6(27), 1(28), 2(30), 1(32)\}$$

$$LJLV_{3b} = [214/204] \{1(18), 3(22), 2(26), 6(27), 1(28), 2(30), 1(32)\}$$

From above tables, it can be observed that both the chains have same LJL values as well as string values and thus are isomorphic.

**Example 3:** Consider two isomorphic kinematic chains [10] having 10 links with 3-dof as shown in Fig. 4.



**Fig. 4: A pair of 10-links, 3-dof isomorphic chains [10]**

Now the string values and LJL values for pair of kinematic chains shown in Fig. 4 are tabulated in Tables 5 and 6.

**Tab. 5: String Values & LJL Values of chain in Fig. 4(a)**

1	2	3	4	5	6	7	8	9	10	LJL Value
33	34	34	34	33	33	34	34	34	34	<b>337</b>
34	38	36	36	22	22	26	26	26	26	<b>292</b>

**Tab. 6: String Values & LJL Values of chain in Fig. 4(b)**

1	2	3	4	5	6	7	8	9	10	LJL Value
34	34	33	34	33	34	33	34	34	34	<b>337</b>
36	36	34	38	22	26	22	26	26	26	<b>292</b>

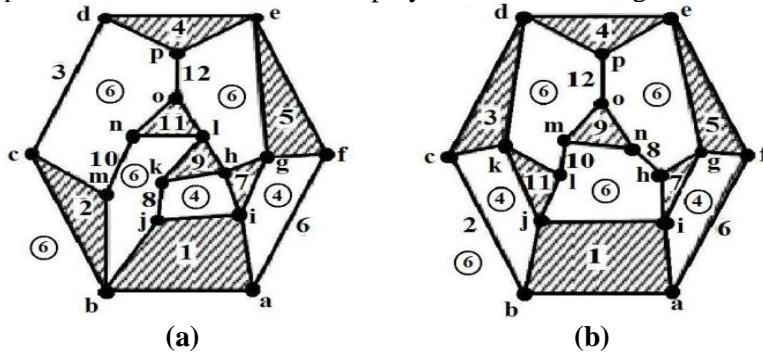
The schematic presentation will be:

$$LJLV_{4a} = [337/292] \{2(22), 4(26), 3(33), 8(34), 2(36), 1(38)\}$$

$$LJLV_{4b} = [337/292] \{2(22), 4(26), 3(33), 8(34), 2(36), 1(38)\}$$

Again, it can be observed that both the chains have same LJL values as well as string values and thus are isomorphic.

**Example 4:** Consider two kinematic chains [25] having 12 links with 1-dof as shown in Fig. 5. The chains possess identical characteristic polynomial of their degree matrices.



**Fig. 5: A pair of 12-link, 1-dof kinematic chains possessing identical characteristic polynomial (Degree Matrix)**

Now the string values and LJL values for pair of kinematic chains shown in Fig. 5 are tabulated in Tables 7 and 8.

**Tab. 7: String Values & LJL Values of chain in Fig. 5(a)**

1	2	3	4	5	6	7	8	9	10	11	12	LJL Value
50	48	48	47	47	49	48	49	47	48	47	48	576
40	36	24	36	32	20	28	20	32	24	36	24	352

**Tab. 8: String Values & LJL Values of chain in Fig. 5(b)**

1	2	3	4	5	6	7	8	9	10	11	12	LJL Value
50	51	47	45	47	51	47	49	49	49	47	48	580
40	20	32	36	32	20	32	24	36	24	32	24	352

The schematic presentation will be:

$$LJLV_{5a} = [576/352] \{2(20), 3(24), 1(28), 2(32), 3(36), 1(40), 4(47), 5(48), 2(49), 1(50)\}$$

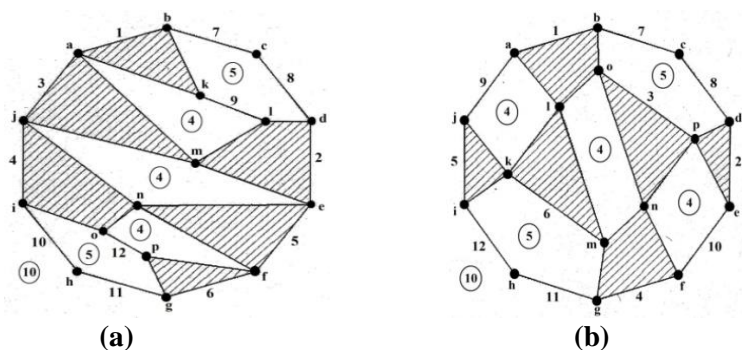
$$LJLV_{5b} = [580/352] \{2(20), 3(24), 4(32), 2(36), 1(40), 1(45), 4(47), 1(48), 3(49), 1(50), 2(51)\}$$

From above tables it can be observed that the string values and LJL values of both the chains

are different from each other and hence are non-isomorphic.

**Example 5:** Consider a pair of kinematic chains having 12 links with 1-dof as shown in Fig.6. The chains possess identical characteristic polynomial of their degree matrices.





**Fig. 6: A pair of 12-link, 1-dof kinematic chains**

Now the string values and LJL values for pair of kinematic chains shown in Fig. 6 are tabulated in Tables 9 and 10.

**Tab. 9: String Values & LJL Values of chain in Fig. 6(a)**

1	2	3	4	5	6	7	8	9	10	11	12	LJL Value
52	49	49	49	49	52	55	53	56	53	55	56	<b>628</b>
38	46	36	46	36	38	30	30	18	30	30	18	<b>396</b>

**Tab. 10: String Values & LJL Values of chain in Fig. 6(b)**

1	2	3	4	5	6	7	8	9	10	11	12	LJL Value
49	52	50	49	52	50	53	54	53	53	53	54	<b>622</b>
46	38	26	46	38	26	30	30	28	28	30	30	<b>396</b>

The schematic presentation will be:

$$LJLV_{6a} = [628/396] \{2(18), 4(30), 2(36), 2(38), 2(46), 4(49), 2(52), 2(53), 2(55), 2(56)\}$$

$$LJLV_{6b} = [622/396] \{2(26), 2(28), 4(30), 2(38), 2(46), 2(49), 2(50), 2(52), 4(53), 2(54)\}$$

From above tables it can be observed that the string values and LJL values of both the chains are different from each other and hence the chains are distinct in nature.

## V. CONCLUSION

Unified quantitative methods, simple and less time consuming, are desirable to compare the chains quantitatively at the conceptual stage without carrying the dimensional synthesis of design for characteristics like static and dynamic behavior, workspace, etc. Most of the methods in structural analysis have been reported to test isomorphism, but the methods reported so far are either based on link-link adjacency, link-joint

adjacency or link-loop adjacency. But all the three important features of kinematic chains viz. link-joint-loop have not been considered for the exact assessment. Keeping in view, quantitative methods are developed to detect isomorphism and to compare chains for the specified task which includes link-joint-loop adjacency.

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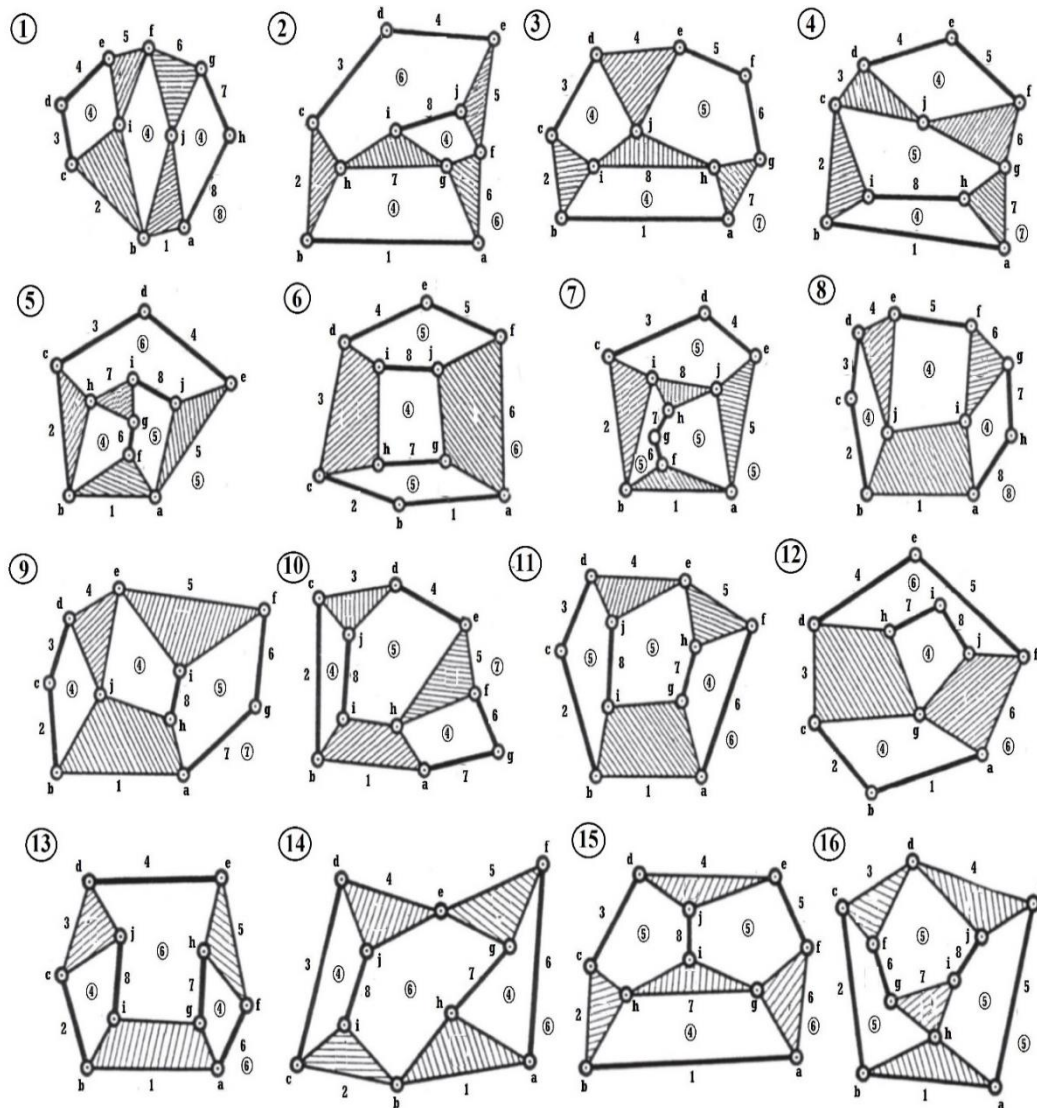
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## Appendix -I

### Schematic Presentation of L/JL Values for 8-Links 1-dof Kinematic Chains

Chain No.	Schematic of L/JL Values
1	[220/224] {4(24), 4(27), 4(28), 4(32)}
2	[212/208] {2(20), 2(24), 4(26), 4(27), 2(28), 2(32)}
3	[217/212] {2(22), 2(24), 1(26), 7(27), 1(28), 1(30), 2(32)}
4	[218/212] {1(18), 3(22), 6(27), 2(28), 4(32)}
5	[214/204] {1(18), 3(22), 2(26), 6(27), 1(28), 2(30), 1(32)}
6	[230/204] {2(18), 4(22), 4(28), 2(29), 2(30), 2(40)}
7	[216/200] {4(20), 8(27), 4(30)}
8	[228/224] {5(24), 1(26), 2(28), 4(29), 1(30), 2(32), 1(40)}
9	[225/212] {1(18), 2(22), 2(24), 2(27), 4(28), 1(29), 2(30), 1(32), 1(40)}
10	[225/212] {1(18), 3(22), 1(24), 1(26), 5(28), 1(29), 1(30), 2(32), 1(40)}
11	[221/204] {1(18), 2(20), 2(22), 5(27), 2(28), 2(30), 1(32), 1(40)}
12	[228/208] {4(20), 2(24), 2(27), 4(28), 2(31), 2(40)}
13	[222/208] {4(20), 1(24), 1(26), 2(27), 4(28), 1(30), 2(32), 1(40)}
14	[212/208] {4(20), 4(26), 4(27), 4(32)}
15	[213/204] {2(20), 2(22), 3(26), 5(27), 1(28), 2(30), 1(32)}
16	[212/200] {4(20), 4(26), 4(27), 4(30)}

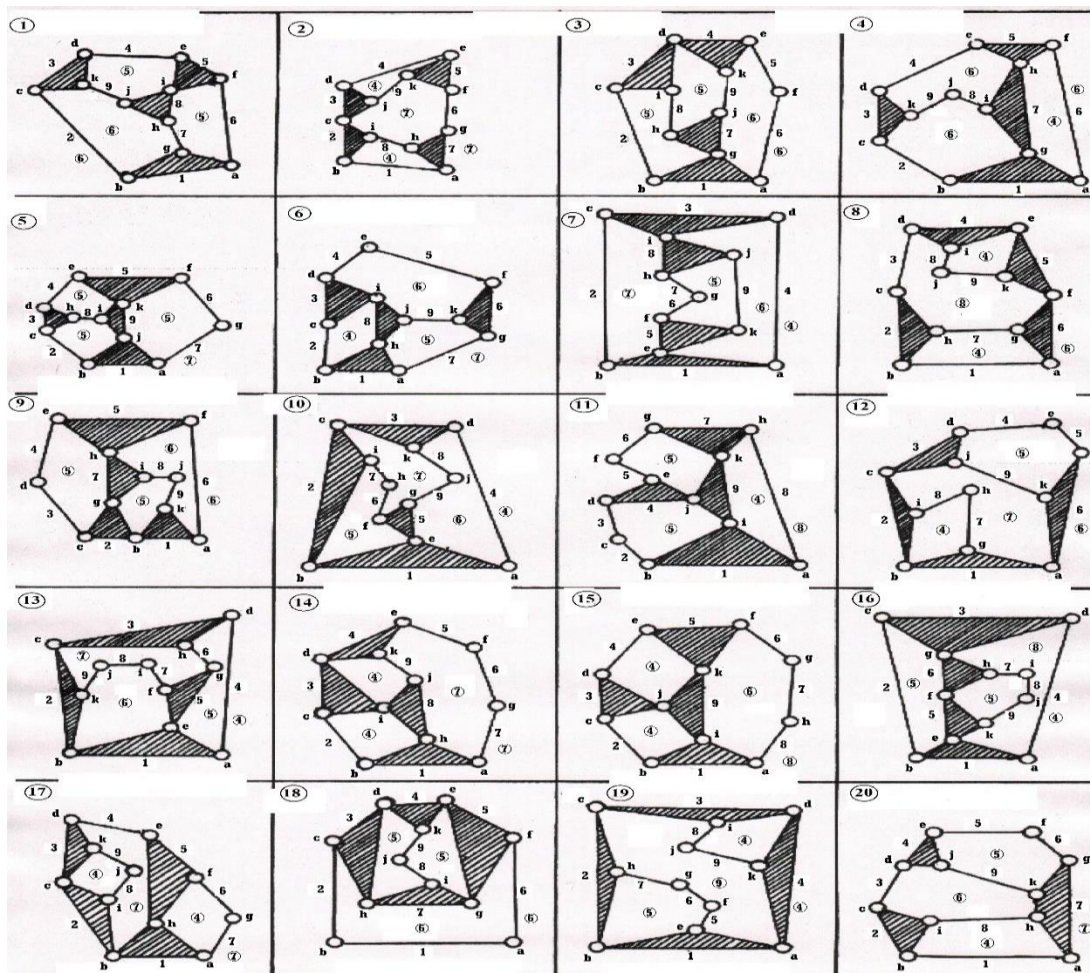


**Appendix – II**

**Schematic Presentation of LJL Values for 9-Links 2-dof Kinematic Chains**

Chain No.	Schematic of LJL Values
1	[270/244] {4(22), 1(24), 9(30), 2(32), 2(34)}
2	[280/260] {4(22), 1(28), 1(30), 6(31), 2(32), 4(36)}
3	[268/244] {1(20), 2(22), 2(24), 2(29), 7(30), 2(32), 2(34)}
4	[269/248] {1(20), 4(24), 3(29), 4(30), 2(31), 3(32), 1(36)}
5	[273/248] {1(20), 4(24), 7(30), 3(31), 3(34)}
6	[274/252] {2(22), 1(24), 2(26), 1(29), 4(30), 5(31), 1(32), 1(34), 1(36)}
7	[273/252] {1(20), 2(22), 2(24), 1(29), 4(30), 4(31), 2(34), 2(36)}
8	[278/264] {1(20), 3(24), 1(28), 1(29), 3(30), 2(31), 2(32), 1(33), 4(36)}
9	[271/244] {4(22), 1(24), 1(29), 6(30), 2(31), 2(32), 2(34)}
10	[272/252] {1(20), 2(24), 2(26), 8(30), 2(31), 1(32), 1(34), 1(36)}
11	[277/260] {1(24), 4(26), 1(28), 2(30), 7(31), 2(34), 1(36)}
12	[276/252] {4(22), 1(24), 5(30), 2(31), 2(32), 2(34), 2(36)}
13	[275/252] {1(18), 1(24), 3(26), 6(30), 3(31), 2(32), 1(34), 1(36)}

<b>14</b>	<b>[278/260]</b> {2(22), 3(28), 4(30), 6(31), 1(32), 2(36)}
<b>15</b>	<b>[278/264]</b> {2(24), 4(28), 3(30), 4(31), 3(32), 2(36)}
<b>16</b>	<b>[279/260]</b> {1(18), 1(24), 3(26), 2(30), 6(31), 1(33), 2(34), 2(36)}
<b>17</b>	<b>[283/260]</b> {4(22), 1(28), 1(29), 2(30), 2(31), 2(32), 2(34), 4(36)}
<b>18</b>	<b>[275/244]</b> {2(20), 3(24), 6(30), 2(31), 2(32), 1(33), 2(34)}
<b>19</b>	<b>[284/276]</b> {2(26), 3(28), 2(30), 4(31), 2(33), 3(34), 2(36)}
<b>20</b>	<b>[284/252]</b> {1(20), 2(22), 2(24), 1(26), 1(30), 3(31), 4(32), 1(33), 1(34), 1(36), 1(44)}



contd....

Chain No.	Schematic of L.J.L. Values
<b>21</b>	<b>[288/260]</b> {4(22), 2(28), 2(31), 6(32), 1(34), 2(36), 1(44)}
<b>22</b>	<b>[277/248]</b> {2(20), 4(24), 4(30), 4(31), 1(32), 1(33), 1(36), 1(44)}
<b>23</b>	<b>[283/248]</b> {2(20), 4(24), 4(30), 4(31), 1(32), 1(33), 1(36), 1(44)}
<b>24</b>	<b>[299/258]</b> {1(18), 4(24), 1(26), 1(31), 1(32), 4(33), 4(34), 1(36), 1(50)}
<b>25</b>	<b>[282/252]</b> {1(18), 2(22), 3(26), 2(30), 3(31), 4(32), 1(33), 1(36), 1(44)}
<b>26</b>	<b>[286/252]</b> {4(22), 1(24), 1(26), 1(29), 4(31), 3(33), 2(34), 1(36), 1(44)}
<b>27</b>	<b>[286/264]</b> {1(20), 3(24), 2(28), 1(30), 2(31), 4(32), 2(33), 2(36), 1(44)}
<b>28</b>	<b>[300/248]</b> {2(20), 3(22), 1(26), 1(31), 2(32), 3(33), 2(34), 2(35), 1(36), 1(48)}



29	[287/260] {1(18), 1(24), 4(26), 1(30), 2(31), 4(32), 1(33), 2(34), 1(36), 1(44)}
30	[387/260] {1(18), 5(26), 4(31), 2(32), 3(33), 2(34), 1(44)}
31	[295/260] {2(20), 3(24), 1(26), 1(31), 2(32), 5(33), 1(35), 2(36), 1(50)}
32	[281/252] {4(22), 2(26), 2(30), 5(31), 1(32), 2(33), 1(36), 1(44)}
33	[305/276] {1(18), 4(26), 2(28), 1(31), 1(32), 2(33), 3(34), 2(36), 1(38), 1(62)}
34	[284/248] {2(20), 4(24), 2(30), 4(31), 1(32), 2(33), 1(34), 1(36), 1(44)}
35	[289/276] {4(26), 2(28), 2(31), 4(32), 3(33), 2(36), 1(44)}
36	[288/252] {2(20), 3(24), 1(26), 1(30), 2(31), 3(32), 2(33), 2(34), 1(36), 1(44)}
37	[288/244] {1(20), 4(22), 2(24), 2(31), 5(32), 2(33), 2(44)}
38	[291/252] {1(18), 1(20), 2(24), 3(26), 6(32), 3(33), 2(44)}
39	[292/252] {2(20), 3(24), 2(26), 2(31), 4(32), 3(34), 2(44)}
40	[302/248] {2(20), 3(22), 2(26), 1(31), 1(32), 4(33), 2(34), 1(35), 1(38), 1(56)}

