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The Key Generation Process by 3D-Fuzzy Chaotic Model for Secure Communications

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Abstract

In this work we proposed a novel way in generating the keys for encryption models. This through employing the Knapsack problem and the continuous chaotic map within the Takagi-Sugeno Fuzzy model. The outputs of the Fuzzy chaotic model are used to create the secure ephemeral key. A chosen primitive variable and randomly choice of the initial values add feature to the chaotic system such nonlinearity. Orbit of the discrete points is determined based on the fuzzy chaos system that is unpredictable. On the proposed Knapsack fuzzy chaotic cryptosystem, the keys are generated, and the masked ciphertext are computed based on the super-increasing sequence which is combined with a secure ephemeral key by implementing the Lorenz map.

Keywords: 3D-Fuzzy Chaotic Model, Communications, Lorenz map.

1. INTRODUCTION

The Mamdani model and Takagi-Sugeno (TS) model [1] are fuzzy models which designed through the performance of the fuzzy rule base from fuzzy IF–THEN rules with antecedent part as fuzzy sets and consequent parts as the linear models. The overall fuzzy inference engine are inputs to output relationship of any general nonlinear system in an interesting region.

Furthermore, the chaotic maps and chaotic behavior for dynamics are deterministic by sensitivity to initial conditions. In spite to the infinitesimal changes in the initial conditions, they may be led to an exponential divergence of orbits. The pioneering work of Carroll and Pecora [2], who first worked on the synchronization and led to many works later on two chaotic systems [3][4], with identical oscillations. Many theories in [5], were proposed to achieve the synchronized manner between these two systems, which are called master–slave configuration. That mainly consists of the original chaotic system as a drive system that provide or a driving signal to synchronize another system that is called a response system. Chaotic signals are featured to be broadband, noise like signal, and difficult to unpredictable. So, they could be used with various context for masking information-bearing waveforms. The signal masking or parameter modulation approaches to chaotic communications only with lower level security as stated in [6]. Many works are used the chaotic communications to increase the security [7][8]. In 2006, Zhong Li published his book that entitled by Fuzzy Model Based Chaotic Cryptosystems. It includes the proposition of a TS-fuzzy chaotic models to modulate a cryptosystem performed using the Henon map [9]. In same year, Li [10] presented two ways to create a secure key through the output of the TS-fuzzy chaotic drive system or by any state with the synchronization error approaches zero [11].

In 2011 Kocarev and Lian [12] presented a contribution for emerging the chaos based cryptography. In 2014, Saad M. Darwish and et al [13], proposed the system of the encrypted database that utilized a chaotic encryption based on cellular automata to realize a higher complexity of crypt-analytical attacks, and generate a symmetric key. A fuzzy observer for synchronizing the chaotic keys scheme is employed to enhance key distribution. In 2017 the Brazilian Yuri S. Villas presented four approaches for knapsack-based public-key cryptosystems in SRVB [14]. In 2018, Hamdy M. Mousa [15] proposed iterative (C-GET) to enhance a secured encryption technique and less a predictable.

In this work, the chaos theory concepts are used based on the fuzzy models to create a key for the chaotic cryptosystem. The Lure type discrete-time chaotic systems are represented by TS-fuzzy models that employed the Lorenz map to generate the keys, to generate a super-increasing sequence of the positive real values. A plaintext in cryptosystem will encrypted using the generated super-increasing sequence to get the ciphertext. The ciphertext directly can be computed to be the output of the drive system. Then the ciphertext embedded into a scalar signal that will send into the response system. All values which are used in the drive system and the regenerated sequence are used to decrypt the ciphertext and recover the original message.

The outline of this work includes: Section 2 displays some concepts and definitions like TS-fuzzy model, Chaos, and Lorenz map are explained. In Section 3, The proposed Knapsack Fuzzy Chaotic KFC cryptosystem is discussed as a main point in this work. The algorithms to generate the keys and compute the encryption and decryption are presented. The study case on the KFC cryptosystem based on the Lorenz map has been illustrated in Section 4. Section 5 presents the security considerations on the KFC cryptosystem. Section 6 draws the conclusions.

2. Mathematical Background

This section includes some important facts related to the fuzzy model and chaotic system.

2.1 The TS Fuzzy Model

Takagi and Sugeno propose a fuzzy model that described by fuzzy if-then rules with consequent parts are represented by linear equations. This fuzzy model takes the following form [9][10]:

$$\boldsymbol{Rule}_{TS}(i): IF\ x_1\ is\ A_{i1}, \dots, x_n is\ A_{in}$$
$$THEN\ y_i = c_{i0} + c_{i1}x_1 + \cdots + c_{in}x_n$$

where $i = 1,2,\dots l$, l is a number of if-then rules, $c_{ik}{}'S$, $k = 0,1,\dots,n$ are the consequent parameters, y_i is the output of i^th^ if-then rule, and A_{ik} are fuzzy sets. Mostly, the TS fuzzy models can be represented by all nonlinear dynamical systems to somewhat high precision.

2.1 *The TS fuzzy model for chaotic system*

Two types of the TS fuzzy modeling for the chaotic systems are distinct, the continuous-time and discrete-time TS fuzzy model [9]. This work focuses on the discrete-time type with three dimensions, but firstly it uses the continuous time type. The idea is to use the continuous-time model and then discretize it to the discrete-time model theoretically. It is presented to get a trajectory of a point in the phase space of the chaotic system. The fuzzy chaotic system is explained in the following steps:

1. Suppose a 3-dimensional continuous-time chaotic system.
2. Determine the primitive variables in a system $x_1(t),\ x_2(t),\ x_3(t)$
3. Design the TS-fuzzy model with if-then rules for a primitive variable that gives a nonlinearity in chaotic system such, $x_1(t)$ on interval $[-d, d]$, and evaluate the parameters to get system matrix.
4. Discretize the continuous-time TS-fuzzy model, by suppose $T_s$sec. a sampling time value that discrete the time to get new system matrix for discrete-time version from TS fuzzy model.
5. Calculate table values for iterations of TS system starting with initial values $x_1(0), x_2(0), x_3(0)$.

6. Determine the output values that represent the trajectory for $x_1(t)$ in the phase trajectory.

For a continuous-time TS-fuzzy model, consider a class of continuous-time nonlinear control system with an input term $\dot{x}(t) = f(x(t)) + g(x(t))u(t)$
where $x(t) \in R^3$, $[x_1(t) \ \ x_2(t) \ \ x_3(t)]^T$ is the state vector and $f(x(t)), g(x(t)) \in R^3$ are nonlinear vector function with appropriate dimension defined on $x(t)$, $u(t) \in R^m$ is the control input vector for m is number of nonlinear equations in the system. The TS-fuzzy model here composed as

$\boldsymbol{Plant\ Rule\ i}$: $IF\ x_1(t)\ is\ \Gamma_1^i\ and\ x_2(t)\ is\ \Gamma_2^i\ and\ x_3(t)\ is\ \Gamma_3^i$
$THEN\ \dot{x}(t) = A_i x(t) + B_i u(t)$

For $i = 1, 2, \ldots, q$, where q is a number of rules in the model, Γ_j^i are fuzzy sets for $x_j(t)$ and $j = 1,2,3$. $A_i \in R^{3\times 3}$ is a continuous-time control system matrix, $B_i \in R^m$ is the input matrix. These rules characterize local relation(s) of the chosen system in the state space. The essential feature for TS model is expressing the local dynamics of each fuzzy implication through a linear state-space that modeled by fuzzy blending of the local linear system by appropriate membership functions(MFs).

Now for a discrete-time TS fuzzy model, consider a class of discrete-time nonlinear control system as;

$$x(t+1) = f(x(t)) + g(x(t))u(t)$$

where $(t+1)$ is index of time steps. The discrete-time TS-fuzzy model is constructed by

$\boldsymbol{Plant\ Rule\ i}$: $IF\ x_1(t)\ is\ \Gamma_1^i\ and\ x_2(t)\ is\ \Gamma_2^i\ and\ x_3(t)\ is\ \Gamma_3^i$
$THEN\ x(t+1) = D_i x(t) + E_i u(t)$

For $i = 1, 2, \ldots, q$ .Where $D_i \in R^{3\times 3}$ is a continuous-time control system matrix, $E_i \in R^m$ is the input matrix, $x_1(t), x_2(t), x_3(t)$ are the premise variables which consist of the state values in states space; the MFs with any form since choosing MF in fuzzy sets will change so little values that don't effect on computed values, but generally as triangular . In some applications without input term the discrete-time nonlinear system be as; $x(t+1) = f(x(t))$, where $f(x(t))$ is a nonlinear function with appropriate dimension defined on $x(t)$. The TS-fuzzy model here is composed for the rules as :

$\boldsymbol{Plant\ Rule\ i}$: $IF\ x_1(t)\ is\ \Gamma_1^i\ and\ x_2(t)\ is\ \Gamma_2^i\ and\ x_3(t)\ is\ \Gamma_3^i$
$THEN\ x(t+1) = D_i x(t) + b_i(t)$

For $i = 1, 2, \ldots, r$, $D_i \in R^{3\times 3}$ is a system matrix with, and $b_i(t) \in R^2$ denotes the bias term.

2.2.1 *Convert the Continuous-Time TS Fuzzy Model into a Discrete-Time*

In some applications, it requires to convert the continuous model into the discrete model through discretization such as continuous-time for Rossler's system Chua's circuit [16] and chaotic Lorenz system [9]. In this work, we converted the continuous-time for chaotic Lorenz system into the discrete time for chaotic Lorenz system. For this process the following theorem is used. The complete proof is given in [17].

Theorem [9][17]. A continuous-time TS-fuzzy model with a continuous-time plant rule i

$\boldsymbol{Plant\ Rule\ i}$: $IF\ x_1(t)\ is\ \Gamma_1^i\ and\ \ldots\ and\ x_n(t)\ is\ \Gamma_n^i$
$THEN\ \dot{x}(t) = A_i x(t) + B_i u(t)$

can be converted into the discrete-time TS fuzzy model with a discrete-time plant rule *i*

$\boldsymbol{Plant\ Rule\ i}$: $IF\ x_1(t)\ is\ \Gamma_1^i\ and\ \ldots\ and\ x_n(t)\ is\ \Gamma_n^i$
$THEN\ x(t+1) = D_i x(t) + E_i u(t)$

where $D_i = exp(A_i T_s) = I + A_i T_s + A_i{}^2 \frac{T_s{}^2}{2!} + \cdots$, and $E_i = \int_0^{T_s} exp(A_i \tau) B_i d\tau = (D_i - I) A_i{}^{-1} B_i$,
and T_s is the sampling time.

2.3 *Basic Facts of the Chaos Theory*

Chaos is aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions. Many chaotic maps are presented [18], these maps are Logistic map, Henon map, Lorenz map [18][19], Lozi map [20]. This work focuses on the Lorenz map that is explained as follows:

Definition 2.3.1. (Lorenz Map). Lorenz map [19] is a chaotic map with 3-dimensions that is defined by

$$\dot{x_1} = -ax_1 + ax_2$$
$$\dot{x_2} = rx_1 - x_2 - x_1x_3$$
$$\dot{x_3} = x_1x_2 - bx_3$$

where a and b are the control parameters. x_1, x_2, and x_3 are the chaotic system variables. This map known as Lorenz system. Normal values for (a, r, b) are $(10, 28, 8/3)$. A chaotic behavior start revolving alternately about two repelling equilibrium points at $(\pm\sqrt{72}, \pm\sqrt{72}, 27)$.

3. The 3-Dimension Knapsack Fuzzy Chaotic Cryptosystem (3D-KFCC)

An easy Knapsack problem is a problem with inputs that are the sets of positive integers as super-increasing sequences. For more details about the Knapsack cryptosystem, one can see [11]. Alternative version of the Knapsack cryptosystem has been proposed in this work. This version employed the fuzzy model, especially TS fuzzy model applied using the Lorenz chaotic map that defined previously, and a positive real valued super-increasing sequence.

3.1 The 3D-KFCC Key Generation Process

Using first component in vector variable in the nonlinear terms $x(t)=[x_1(t)\ x_2(t)\ x_3(t)]^T$, namely $x_1(t)$, to be the output that generates a secure ephemeral key $k(t)$. That is, user computes the key $k(t)$ by $k(t) = [k(t-0)k(t-1), \dots, k(t-j+1)] = [x_1(t-0)x_1(t-1), \dots, x_1(t-j+1)]$

Now, a super-increasing sequence S_i, for i =1, 2,..., l, is generated through the following computations: $S_1(t) = |k(t)| + \tau$, and $S_j(t) = \sum_{i=1}^{j-1} S_i(t) + |k(t-j+1)| + \tau$,For j=2, 3, 4, …, l and $\tau > 0$. The super-increasing sequence S $=\{S_1, S_2, \dots, S_l\}$ computed secretly. Also a first user computes $S_1+S_2+\dots+S_l = T$ and he/she chooses $N > T$. She/he also selects W, where W in [2, N-1] and $gcd(W,N) = 1$. In the other words, W and N are relatively prime ensuring element W has multiplicative inverse $modN$. First user computes public hard knapsack $B = \{b_1, \dots, b_l\}$, where $b_i = WS_i(modN)$, for $i = 1,2,\dots,l$. Finally, he/she keeps (S, W, N) as private key and B as public key.

4. 3D-Fuzzy Chaotic model

The chaotic Lorenz system for continuous-time is defined previously in Definition (2.3.1). The system with parameters values are: a=10, b=8/3 and r=28, so the system is given by

$$\dot{x_1} = -10x_1 + 10x_2$$
$$\dot{x_2} = 28x_1 - x_2 - x_1x_3$$
$$\dot{x_3} = x_1x_2 - 8/3x_3.$$

The equivalent fuzzy model constructed by

$\boldsymbol{Plant\ Rule\ i}$: $IF\ x_1(t)\ is\ \Gamma_1^i\ THEN\ \dot{x}(t) = A_i x(t)$. Thus, the rules are

$\boldsymbol{Plant\ Rule 1}$: $IF\ x_1(t)\ is$ about -d $THEN\ \dot{x}(t) = A_1 x(t)$,

$\boldsymbol{Plant\ Rule 2}$: $IF\ x_1(t)\ is$ " *about d* " $THEN\ \dot{x}(t) = A_2 x(t)$,

where *"about d"* and *"about -d"* are fuzzy sets corresponding to Γ_1^1, Γ_1^2 in the rules system respectively.

For values of $x_1(t)$, in the interval $[-d, d]$ with d=30, the fuzzy sets defined with the triangular MFs by

"about -d" = $\Gamma_1^1(x_1(t)) = \frac{x_1(t)-(-d)}{d-(-d)} = \frac{x_1(t)+d}{2d}$

, and *"about d"*= $\Gamma_1^2(x_1(t)) = \frac{d-x_1(t)}{d-(-d)} = \frac{d-x_1(t)}{2d}$.

Since $-d, d$ are controlled the nonlinear terms in the system matrices which are given by

$$A_1 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -d \\ 0 & d & -\frac{8}{3} \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & d \\ 0 & -d & -\frac{8}{3} \end{bmatrix}$$

The discretization of the continuous-time chaotic Lorenz system depending on a previous theorem which is given by

$\boldsymbol{Plant\ Rule\ i}$: $IF\ x_1(t)\ is\ \Gamma_1^i\ THEN\ x(t+1) = D_i x(t)$

A bias term considers as a zero vector, the rules:

$\boldsymbol{Plant\ Rule 1}$: $IF\ x_1(t)\ is$ " *about -d* " $THEN\ x(t+1) = D_1 x(t)$

$\boldsymbol{Plant\ Rule 2}$: $IF\ x_1(t)\ is$ " *about d* " $THEN\ x(t+1) = D_2 x(t)$

precisely could be

$\boldsymbol{Plant\ Rule 1}$: $IF\ x_1(t)\ is$ “ *about -d* “

$$THEN \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \end{bmatrix} = D_1 \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$\boldsymbol{Plant\ Rule2}$: $IF\ x_1(t)\ is\ "\ about\ \ d\ "$

$$THEN \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \end{bmatrix} = D_2 \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

where the matrices ; $D_1 =$

$$\begin{bmatrix} 1-aT_s & aT_s & 0 \\ rT_s & 1-T_s & -dT_s \\ 0 & dT_s & 1-bdT_s \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1-aT_s & aT_s & 0 \\ rT_s & 1-T_s & dT_s \\ 0 & -dT_s & 1-bdT_s \end{bmatrix}.$$

The values for T_s supposed to be approvable with the chosen that is employing in this work. We take $T_s = 0.003$ sec. with simulation calculations for $x_1(t) \in [-30,30]$, So the matrices will be

$$D_1 = \begin{bmatrix} 0.97 & 0.03 & 0 \\ 0.084 & 0.997 & 0.09 \\ 0 & -0.09 & 1.008 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.97 & 0.03 & 0 \\ 0.084 & 0.997 & -0.09 \\ 0 & 0.09 & 1.008 \end{bmatrix}.$$

The experiments are implemented staring with different initial values and continued until 40 times as shown in Table (2) that refers to the points in the phase trajectory instead of the continuous connected curve.

4.1 The Experiment

The TS-fuzzy model employed using the Lorenz map with initial conditions $[-10 \quad 10 \quad 10]$ for vector state. This model iterated for 40 iterations to extract the secure key $k(t)$ from the state trajectory of the state $x_1(t)$. The value for τ assumed 18 since this value has many factors $\{1, 2, 3, 6, 9, 18\}$.

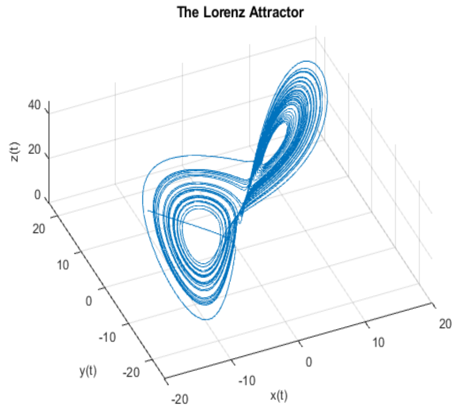


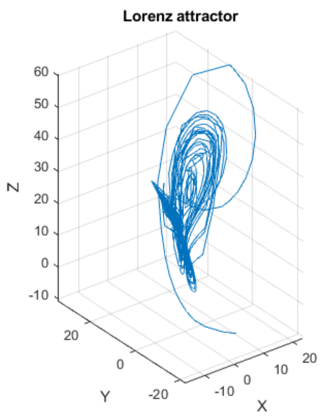
Figure 1. *The Lorenz Chaotic Map.*

TABLE : *TS chaotic fuzzy model values for continuous chaotic map*

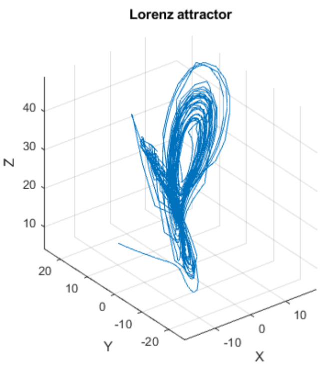
Time t	$x_1(t)$	$x_2(t)$	$x_3(t)$	$k(t)+\tau$
0	-10	10	10	8
1	-9.4	10.03	9.18	8.6
2	-8.8171	10.03651	10.15614	9.1829
3	-8.2514917	10.17981667	11.14067502	9.748508
4	-7.698552449	10.45881267	12.14598392	10.30145
5	-7.153831495	10.87389638	13.18444493	10.84617
6	-6.612999659	11.42695289	14.26857117	11.387
7	-6.071801083	12.12135146	15.41114549	11.9282
8	-5.526006506	12.96195921	16.62535629	12.47399
9	-4.971367535	13.95517085	17.92493547	13.02863
10	-	15.10895	19.32430	13.59643

	4.403571383	466	033	
11	-3.818195602	16.43291483	20.83870065	14.1818
12	-3.210662289	17.93837071	22.48437259	14.78934
13	-2.576191299	19.6384535	24.27870094	15.42381
14	-1.909751955	21.54822116	26.24039136	16.09025
15	-1.206012762	23.68479255	28.38965439	16.79399
16	-0.459288602	26.067502	30.74840296	17.54071
17	0.336515116	28.71807552	33.34046536	18.33652
18	1.187961928	31.66083044	36.19181588	19.18796
19	2.102147983	34.92290018	39.33082515	20.10215
20	3.086770549	38.53448617	42.78853277	21.08677
21	4.150202018	42.52913939	46.59894478	22.1502
22	5.301570139	46.94407397	50.79935889	23.30157
23	6.550845254	51.82051594	55.43072042	24.55085
24	7.908935375	57.20409023	60.53801261	25.90894
25	9.387790021	63.14524967	66.17068484	27.38779
26	11.0005	69.69974	72.38312	29.00051

	1381	992	279	
27	12.76149089	76.92917488	79.23516526	30.76149
28	14.68652141	84.90151746	86.79267232	32.68652
29	16.79297129	93.69182122	95.12815027	34.79297
30	19.09993679	103.3828889	104.3214394	37.09994
31	21.62842535	114.0660644	114.4604709	39.62843
32	24.40155453	125.8420964	125.6421005	42.40155
33	27.44477078	138.8220897	137.9730259	45.44477
34	30.78609035	153.1285565	151.5707982	48.78609
35	34.45636433	168.8965743	166.5649347	52.45636
36	38.48957063	186.2750633	183.0981458	56.48957
37	42.92313541	205.4281951	201.3276867	60.92314
38	47.7982872	226.5369457	221.4268458	65.79829
39	53.16044696	249.8008071	243.5865856	71.16045
40	59.05965776	275.439675	268.017351	77.05966



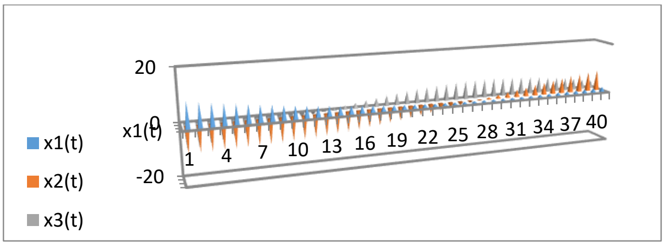
a



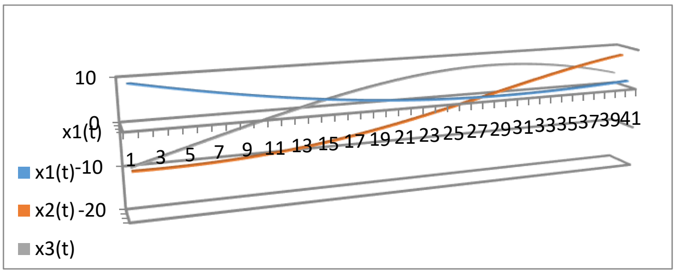
b

FIGURE 2: *(a) Lorenz attractor with initial value [10 -10 -10]. (b) Lorenz attractor with initial value [-10 10 10]*

These figures were drawn by using MATHLAB R2018b, for iteration 40, and episode= 0.10000001, which represent the Ordinary differential equation solver precision in drawing the figure.



(a)



(b)

FIGURE 3: *Trajectories for x_1 (t),x_2 (t),x_3 (t) in two different manner*

A real values super-increasing sequence as keys are computed with five terms after 40 iterations by

$S = \{8, 8.6, 16.79399, 34.79297, 71.16045\}$

,

It can see, the system implemented with initial values for state vector as $[\,10 \quad -10 \quad -10]$, then a superincreasing sequence is not obtained even after 40 iterations.

5. Conclusions

The proposed Knapsack fuzzy chaotic cryptosystem applied the Lorenz map that uses the TS-fuzzy model to generate a secret key to given another version of fuzzy chaotic cryptosystem. The super-increasing sequence with positive real values added more complicated in computations and predictions. The study case implemented on continuous Lorenz map discrete theoretically by random time sample. A fuzzy model is implemented for 40 iterations on the chaotic system. The initial values assumed in two state vectors [-■(10&10&10]) and [■(10&-10&-10)]. All values and work performed on 1st vector while don't work with 2nd after 40 iterations. Resulted trajectory for a primitive variable are used to generate secret key by choosing a super-increasing sequence from data. A good results and flexibility for changing the supposed values made the system behavior is unpredictable. The security of the proposed KFC cryptosystem is determined through many points: Employing continuous chaotic map and then discrete it, using a random sample time in discretize, choosing the parameters values for chaotic map with chaotic behavior, assume initial values randomly from closed interval in state space, choosing chaotic map is unpredictable, finding a super-increasing sequence from the trajectory (orbit) of points, increasing of the iterations and when the trajectory has more than one super-increasing.

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