Some chaotic properties for g-non autonomous systems

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Abstract:
In this work , we study some chaotic properties of general non autonomous discrete systems : we shall discuss this properties transitive sets, weakly mixing sets in space more general is g-non autonomous discrete systems .

INTRODUCTION

In [3] Liu and sun are studied weakly mixing set and transitive set In space called non-autonomous discrete system .Let N be natural number and Y be a topological space , $h_n : Y \to Y$, for all $n \in N$ be continuous maps , $h_{1,\infty}$ denoted the Sequences $< h_1, h_2, \dots, h_m, \dots >$ and $h_1^m = h_m \circ h_{m-1} \circ \dots \circ h_2 \circ h_1$ .in this paper we introduced g- non autonomous discrete system . Let $h_{n,\infty}$ be denoted the sequences $< h_m, h_{m-1}, \dots, h_n, \dots >$, for all $n > m \in N$. The pair (Y ,$h_{n,\infty}$) is called g- anon autonomous discrete system and define by $h_n^m = h_m \circ h_{m-1} \circ \dots \circ h_n$, $\forall$ n>m$\in$ N . h_n^0 = identity on Y . So when $h_{n,\infty}$ is constant sequences <h ,h ,…, h ,….> then the pair (Y ,$h_{n,\infty}$) is called classical discrete dynamical system .So the orbit initiated from $y \in Y$ under $h_{n,\infty}$ is define by the set $\beta(Y, h_{n,\infty}) = \{ y , h_n(y), h_n^2(y), \dots, h_n^m(y), \dots \}$. Let (Y ,h) be topological dynamical system . (Y ,h) is topological transitive if for any open subsets $M, W \neq \emptyset$ $\exists$ $n \in N$ $\ni$ $h^n(M) \cap W \neq \emptyset$.(Y ,h) is called topological mixing if for any open subsets of Y $M, W \neq \emptyset$ $\exists$ $D \in N$ $\ni$ $h^n(M) \cap W \neq \emptyset$, $\forall n \in N$, $n \geq D$.(Y ,h) is called topological weakly mixing if for any open subsets of Y $M_i, W_i \neq \emptyset$, when i = 1,2 $\exists n \in N \ni h^n(M_i) \cap W_i \neq \emptyset$ when i = 1,2 .

Although Shi and Chen [4] and in [5] also in [1] they studied chaos in space called non-autonomous discrete system but in [2] Devaney defined strong chaos with three conditions on any spaces : transitivity , sensitivity and density of periodic point. in this search we discussed some chaotic properties for example weakly mixing and transitive and other for space is more general than introduced Liu and sun in [3] .

Now : Let B be closed subset of Y and it is contain at least two element . B is called weakly mixing set of Y if and only if for any $k \in N$, any $\emptyset \neq M_i$ open set , i=1,..,k of Y
And $\emptyset \neq W_i$ open subset of B ,i=1,…,k with $B \cap M_i \neq \emptyset$, $\exists$ $r \in N$ $\ni h^r(W_i) \cap M_i \neq \emptyset$, when i = 1,…,k .

So (Y ,h) is called partial weak mixing if the space Y contain a weakly mixing subset and B is transitive set of (Y ,h) if for any $\emptyset \neq W^B$ open subset of B and $M \neq \emptyset$ open subset of Y with $B \cap M$ is non-empty $\exists$ $n \in N$ $\ni$ $h^n(W^B) \cap M$ is non-empty .

Hint : we will shorten the sentences weakly mixing , transitive set , transitive point , open set , regular closed set ,topologically transitive and g-non autonomous discrete system by W.M.S , T.S , T.P , O.S , R.C.S , T.T and g-NADS respectively.

Preliminaries :

In the present paper int (B) and cl (B) denoted interiors and the closure of the set B respectively . h_n^m denoted $h_m \circ h_{m-1} \circ \dots \circ h_n$, for all $n > m \in N$. We define $(h_k)^n = h_k \circ h_k \circ \dots \circ h_k$ n-time, $\forall$ k,n $\in$ N . g-NADS (Y ,$h_{n,\infty}$) is point transitive if $\exists$ a point $y \in Y \ni$ cl $\beta(y, h_{n,\infty})$ = Y , and y called T.P of (Y, $h_{n,\infty}$) . So (Y, $h_{n,\infty}$) called T.T when we find open sets W, M of Y $\exists$ $r \in N \ni h_n^r(W) \cap M \neq \emptyset$.

(Y, $h_{n,\infty}$) is said to be weakly mixing if for any non-empty open sets W_i and M_i of Y, for i = 1,2 if accur $r \in N \ni h_n^r(W_i) \cap M_i \neq \emptyset$

Published: May/ 2022

Definition 2.1 :

Assume that (Y, $h_{n,\infty}$) g-NADS. B subset of Y is said to be invariant if $h_n^m(B)$ *is* subset of B , ∀ n>m ∈ N.

Definition 2.2 :

Assume that (Y,$h_{n,\infty}$) g-NADS and ∅ ≠B closed subset of Y .B is a T.S of (Y,$h_{n,\infty}$)when we find non-empty open set W^B of B and anon-empty O.S M of Y with

B ∩ $M \neq \emptyset$∃ n ∈ N ∋ h_n^m (W^B) ∩ $M \neq \emptyset$.

Remark :

Y is T.S of (Y, $h_{n,\infty}$) . this statement is true if (Y,$h_{n,\infty}$) is T.T .

Definition 2.3 :

Assume that (Y, $h_{n,\infty}$)g-NADS and B non-empty closedsubset of Y contain at least two element . B is W.M set of (Y,$h_{n,\infty}$) if ∀ k ∈ N and $\emptyset \neq W_1^B , W_2^B ,\dots, W_k^B$are open set of B, $\emptyset \neq M_1 ,M_2 ,\dots, M_k$are open subsets of Y with B ∩ $M_i \neq \emptyset$, i=1,2 ,..,k ∃ n>m ∈ N ∋ h_n^m (W_i^B) ∩ $M_i \neq \emptyset$, For each $1 \leq i \leq k$.

By definition of the transitive , W.M.S of g- anon autonomous discrete system we obtained the followingremarks :

1. If a ∈ Y is T.P of (Y, $h_{n,\infty}$) ,Then $\{a\}$ is T.S of (Y, $h_{n,\infty}$) .
2. If B is W.M set of (Y, $h_{n,\infty}$) , Then B is T.S of (Y , $h_{n,\infty}$) .

For example : Let

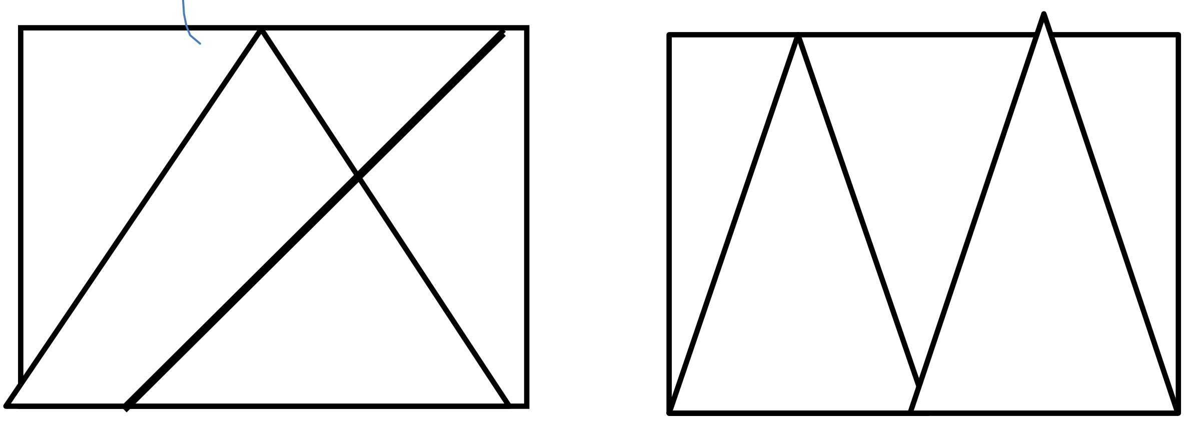
$$T_n(y) = \begin{cases} \frac{2n}{n-2} y & \text{if } 0 \leq y \leq \frac{n-2}{2n} \\ 1 & \text{if } \frac{x-2}{2n} \leq y \leq \frac{n+2}{2n} \\ \frac{2n}{n-2}(1-y) & \text{if } \frac{n+2}{2n} \leq y \leq 1 \end{cases} \quad \text{when } n=3,4,\dots$$

and T_1= T_2 = id the identity map on [0,1] .

Observe that the given sequence converges uniformly to tent map .

$$T(y) = \begin{cases} 2y & \text{if } 0 \leq y \leq 0.5 \\ 2-2y & \text{if } 0.5 \leq y \leq 1 \end{cases}$$

We can show that the interval from 0 to 0.5 is a T.S of (Y ,$h_{n,\infty}$)



Thefirst iterate The second iterate

Definition 2.4 :

Assume that (Y,r) a topological space, $\emptyset \neq$B of Y . B is R.C.S of Y when B=cl(int (B)). We can show that B is a R.C.S if and only if int (w^B) $\neq \emptyset$,for any $W^B \neq \emptyset$ of B(Liu and sun,2014).

Definition 2.5 :

Assume that (Y ,r) be a topological space, W and M be two non-empty subsets of Y . M is dense in W if W is subset of cl (W $\cap M$)(Liu and sun,2014)

We can prove that M is dense in W if and only if $W^B \cap M \neq \emptyset$ for any
$\text{non} - \text{empty } O.S\ W^B$ of B .

3 - Main results :

In g-NADS , we introduced some properties T.S and W.M.S.

Proposition 3 .1 :

Assume that (Y, $h_{n,\infty}$) g-NADS with $\emptyset \neq B\ closed\ set\ of\ Y$ be a . The conditions listed below are equivalent :

1 . B is T.S of (Y,$h_{1,\infty}$) .

2 . B is T.S of (Y, $h_{n,\infty}$) .

3 . Let $\emptyset \neq W^B open\ subset\ of\ B$ and $\emptyset \neq$M open subset of Y in the company of B $\cap M \neq \emptyset$ then accur n>m $\in N \ni W^B \cap (h^m_n)^{-1}$(M) $\neq \emptyset$.

4 . Assume that$\emptyset \neq$M O.S of Y with $M \cap B \neq \emptyset\ then\ \cup_{n,m\in N} (h^m_n)^{-1}(M)$ is dense in B .

Proof :

(1→ 2) . we use the definition . the result hold .

(2→ 3) .Let B transitive set of (Y, $h_{n,\infty}$) . by definition of T.S then for any choice of a $\emptyset \neq W^B\ O.S$ of B and $\emptyset \neq$ M O.S of Y with B $\cap M \neq \emptyset$ there exists n $\in N \ni h^m_n(W^B) \cap M \neq \emptyset$. Since $h_n : Y \to Y$, For any n$\in N$ continuous maps and B closed set of Y Then h^{-1} exists so $h^m_n(W^B \cap (h^m_n)^{-1}(M)) = h^m_n(W^B)\ h^m_n(h^m_n)^{-1}(M) = h^m_n(W^B) \cap M$ but $h^m_n(W^B) \cap M \neq \emptyset$, we take h^{-1}_n two side $(h^m_n)^{-1} h^m_n(W^B \cap (h^m_n)^{-1}(M)) \neq h^{-1}_n(\emptyset) = \emptyset$. Then $W^B \cap (h^m_n)^{-1}$ (M) $\neq \emptyset$.

(3 → 4) .Assume that $\emptyset \neq W^B\ O.S$of B and $\emptyset \neq$M O.S of Y with B$\cap M \neq \emptyset$. by assumption there exist n>m $\in N \ni W^B \cap (h^m_n)^{-1}(M) \neq \emptyset$ by Definition 2.5 then $(h^m_n)^{-1}$ (M) is dense in B and $W^B \cap \cup (h^m_n)^{-1}(M) \neq \emptyset$. Then $W^B \cap \cup (h^m_n)^{-1}(M) = \cup_{n,m\in N} (W^B \cap (h^m_n)^{-1}(M)$ Then $\cup (h^m_n)^{-1}$ (M) is dense in B .(4→ 1) . Let $W^B$is any non-empty open set of B and $\emptyset \neq$M be open set of Y such that B $\cap M \neq \emptyset$ .by assumption $\cup_{n,m} (h^m_n)^{-1}(M) is\ dense\ in\ B$. by Definition 2.5 then$W^B \cap \cup_{n,m} (h^m_n)^{-1}(M) \neq \emptyset$.Then accrue n> m $\in N$
$\ni W^B \cap (h^m_n)^{-1}(M) \neq \emptyset$. It flowed $h^m_n(W^B \cap ((h^m_n)^{-1}(M)) \neq h^m_n(\emptyset) = \emptyset\ so$
$h^m_n(W^B) \cap h^m_n((h^m_n)^{-1}(M)) \neq \emptyset\ , then\ h^m_n(W^B) \cap M \neq \emptyset$. Then the result is hold ■

By this theorem , we obtained the following corollary :

Corollary 3-1 :

The following statement are equivalent if (Y , h)is a classical dynamical system and $\emptyset \neq$ B is closed set of Y :

1. The set B is transitive of (Y, h) .
2. Let $\emptyset \neq W^B$ be open subset of B and $\emptyset \neq$M open subset of Y with B $\cap M \neq \emptyset$, then accur n $\in$ N in the company of $W^B \cap h^{-n}(M) \neq \emptyset$.
3. Let M be anon- empty open set of Y with B$\cap M \neq \emptyset$, Then $\cup h^{-n}(M)$ is dense in B , ∀ n$\in N$.

Proposition 3-2 :

Assume that Ybe atopological space ,and $\emptyset \neq$B a closed subset of Y. Then the statement that below are equivalent :

1. B is T.S of (Y,$h_{1,\infty}$) .
2. B is T.S of (Y, $h_{n,\infty}$).
3. If (Y , d) is matric space . Let a , x $\in B$ and $\varepsilon , \delta > 0$ $then$ $\exists$ $n \in N \ni$ B$\cap$ $S(a,\varepsilon) \cap (h_n^m)^{-1}S(x,\varepsilon) \neq \emptyset$.
4. If (Y , d) is matric space . Let a , x $\in B$ and $\varepsilon > 0$ $then$ $\exists$ $n \in N \ni$
$B \cap S(a,\varepsilon) \cap (h_n^m)^{-1}S(x , \varepsilon) \neq \emptyset$.

Proof:

(1→ 2) .we using the definition .the result hold .

(2→ 3) .Let B be transitive set of (Y , $h_{n,\infty}$), so by definition of transitive set the result hold .

(3→ 4) .clear.

(4→ 1) .Let $\emptyset \neq W^B$ O.S of B and $\emptyset \neq$M O.S of Y with B $\cap M \neq \emptyset$. Since (Y , d) matric space then there exists a ,x $\in B$ and $\varepsilon > 0 \ni$
S(a ,ε) and $S(a , \varepsilon)$ are exists so S(a ,ε) subset of B and S(x,ε) $subset$ of M so $B \cap S(a,\varepsilon) = S(a,\varepsilon)$ $subset$ of W^B is open set of B .by assumption
(B $\cap$ $S(a,\varepsilon)$) $\cap$ $(h_n^m)^{-1}S(x,\varepsilon)) \neq \emptyset$.Then $W^B \cap (h_n^m)^{-1}(M) \neq \emptyset$,thenB is T.S of
(Y , $h_{n,\infty}$) ■

Proposition 3.3 :

Let (Y ,$h_{n,\infty}$) be g-NADS and B is T.S of(Y,$h_{n,\infty}$) . Then

1. M is dense in B if$\emptyset \neq$ M O.S of Y satisfying $\emptyset \neq$B$\cap$M and $(h_n^m)^{-1}(M)$ subset of U , for all n>m are natural number .
2. if H is a closed invariant of Y and H is a subset of Y , then H=B or E is nowhere dens
 in B .
3. B if B is a R.C.S of Y ,Then $\cup_{n,m\in N}$ $h_n^m(B)$ is dense in B .

Proof :

1 . By hypothesis , We get$\cup_{n,m\in N}$ $(h_n^m)^{-1}(M)$ is subsetofM,so byproposition3-1 $\cup_{n>m\in N}$ $(h_n^m)^{-1}(M)$ is dense in B .Then wehave M is dense in B .

2.By hypothesis (2) then H $\neq B$ and M=Y-U is an open set of Y and M$\cap$ B $\neq \emptyset$. Since H is an invariant then $h_n^m(H)$ subset of H , $\forall$ $n > m \in N$ but H is closed and h_n is continuous then h^{-1}exists so $(h_n^m)^{-1}(E)$ $subset$ of E and
$(h_n^m)^{-1}(M) = (h_n^m)^{-1}(Y - H) = (h_n^m)^{-1}(Y) - (h_n^m)^{-1}(H)$ subset of Y-H=M,for all n>m $\in N$. Hence $(h_n^m)^{-1}(M)$subset of M, for all n>m$\in$ N by 1 M is dense in B .Hence H is nowhere dense .

3- Let $\emptyset \neq W^B$ be O.S of B .Then int(W^B) $\neq \emptyset$ .since B is regular closed (B$\neq \emptyset$) then int(B)$\neq \emptyset$ .since B is T.S of (Y,$h_{n,\infty}$) .Then any n>m$\in$ $N \ni (h_n^m)(int(B)) \cap int(W^B) \neq \emptyset$, but W^B is non-empty open set of B then int (W^B) subset of W^B and int(B) subset of B so $h_n^m(B) \cap W^B$ is non-empty Then $h_n^m(B)$is dense in B , for all n>m $\in N$. $Hence$ $\cup_{n>m\in N}$ (h_n^m) is dense in B■

Theorem 3.1 :

Assume that(Y,$h_{n,\infty}$) g-NADS and $\emptyset \neq$B be a closed invariant set of Y .Then B is a T.S of (Y, $h_{n,\infty}$)if and only if (B, $h_{n,\infty}$) is T.T .

Proof

(→)Assume thatW^B,$M^B$are two non-empty open subsets of B ,for a non-empty open subset M^B of B there is M of Y such that W^B= W $\cap B$. Since B is transitive so by definition of transitive set There exist n>m $\in$ N $\ni$ $h_n^m(W^B) \cap M$ is non-empty Moreover B is invariant Then h_n^m (B) is subset of B , $\forall$ $n > m \in N$ Which impels that $h_n^m(W^B)$ subset of B (because W^B subset of B) there for $h_n^m(W^B) \cap B \cap M \neq \emptyset$ then $h_n^m(W^B) \cap$ $M^B \neq \emptyset$ .Then (B, $h_{n,\infty}$) is topological transitive .

($\leftarrow$)Let $\emptyset \neq W^B$ be O.S of B and $\emptyset \neq$M be O.S of Y with B$\cap M \neq \emptyset$. since M is an O.S of Y and B$\cap W \neq \emptyset$. it follows that M$\cap B$ is
Non-empty O.S of B . since (B,$h_{n,\infty}$) T.T there exists n,m$\in N \ni h_n^m(W^B) \cap (M \cap B) \neq \emptyset$) which impels that $h_n^m(W^B) \cap M \neq \emptyset$, For all
n>m $\in N$, then B is transitive set of (Y, $h_{n,\infty}$)■

Theorem 3.2 :

Let (Y , $h_{n,\infty}$) be T.T . Then B is T.S of (Y ,$h_{n,\infty}$) ,when B be regular closed set of Y .

proof :

Let $\emptyset \neq W^B$ be set of B and $\emptyset \neq$ M set of Y with B$\cap$M$\neq \emptyset$. since B is regular closed then by Definition 2.4 we get int (W^B) is open subset of W^B then for all n>m $\in N$ such that h_n^m(int(W^B)) $\cap$ M is non-empty but int (W^B) subset of W^B, then $h_n^m(W^B) \cap M \neq \emptyset$, $\forall\, n > m \in N$.Then B is transitive of (Y,$h_{n,\infty}$)■

By above theorem , we get following corollary :

Corollary 3.2 :

Assume that (Y,$\hbar_{n,\infty}$) g-NADS .Then (Y,$\hbar_{n,\infty}$) is T.T if and only if every non-empty R.C.S of Y is also a T.S of (Y,$\hbar_{n,\infty}$) .

Definition 3.1:

Let (Y,$\hbar_{n,\infty}$) , (Z,$g_{n,\infty}$) be a two g-NADSs and let S : Y$\rightarrow Z$ be continuous map ,g_n(S(y)) = S ($\hbar_n$(y)) . $\forall\, n \in N$ and y $\in Y$.

1. If the map S : Y$\rightarrow Z$ is a surjective , then ($\hbar_{n,\infty}$), ($g_{n,\infty}$) are called topologically semi-conjugate .
2. If the map S : Y$\rightarrow Z$ is a homeomorphism,then($\hbar_{n,\infty}$) , ($g_{n,\infty}$) are said to be topological conjugate

.We take the example :

We assume that$\hbar_n: R \rightarrow R$ with $\hbar_n(x) = nx\ for\ any\ n \in N$ and x $\in R$ Where R is a real line and $g_n: s^1 \rightarrow s^1\ with\ g_n(e^{iP}) = e^{inP}$, forany n $\in N$. Define S : R$\rightarrow S^1$
$by\ R(x) = e^{2\pi ix}$, so S is continuous surjective map and S $\circ\hbar_n = g_n \circ S$,then
(R , $\hbar_{n,\infty}$) and ($S^1, g_{n,\infty}$) are topological semi-conjugate.

Theorem 3.3 :

Let(Y, $\hbar_{n,\infty}$) $and\ (Z, g_{n,\infty})\ be\ two$ g-NADSs and S: $Y \rightarrow Z$ map that is semi-conjugate B non-empty closed subset of Y and S(B) closed subset of Z . Then

1. If B is T.S of (Y, $\hbar_{n,\infty}$) , then S(B) is T.S of (Z, $g_{n,\infty}$) .
2. If B is W.M.S of (Y,$\hbar_{n,\infty}$) and h(B) isn't a singleton , then h(B) is W.M.S of
(Z , $g_{n,\infty}$) .

Proof:

1. Let $W^{S(B)}$ be non-empty open set of S(B) and M be non-empty open set of Y
with S(B) $\cap M$ non-empty. Since S(B) is subspace of Y $\exists$ an open set W of Y $\ni$
$W^{S(B)} = W \cap S(B)$. Also B$\cap S^{-1}(W^{S(B)}) = B \cap S^{-1}(W \cap S(B)) = B \cap S^{-1}(W)$

Hence B$\cap S^{-1}(W^{S(B)})$is an open subset of B . But

$$S(B \cap S^{-1}(W^{S(B)}) = S(B) \cap S\,(S^{-1}(W^{S(B)})) = S(B) \cap W^{S(B)} = W^{S(B)} \neq \emptyset$$

(S(B) subspace of Y so$W^{S(B)}$subset of B) and $W^{S(B)}$ is non-empty then
B$\cap S^{-1}(W^{S(B)})$is non-empty.Then B$\cap S^{-1}(W^{S(B)})$ is non-empty and M $\cap S(B)$is non-empty which hold $S^{-1}(M) \cap M \neq \emptyset$. Since B is transitive set of (Y,$\hbar_{n\,\infty,}$) then $S^{-1}(W^{S(B)}) \cap B \cap (\hbar^m)^{-1}(S^{-1}(M)) \neq \emptyset$ (because $S^{-1}(W^{S(B)} \cap B \neq \emptyset\ and\ (\hbar_n^m)^{-1}(S^{-1}(M)) \neq \emptyset$ .As S is semi-conjugate map .i.e , g_r(S(y))=S($\hbar_r$(y)) , For any r $\in N\ and\ y \in Y$ We take inverse two sided ,then we obtained the following :
$S^{-1}\,(g_r)^{-1}(y) = (\hbar_r)^{-1}S^{-1}(y)\ for\ all\ r \in N\ and\ y \in Y$ Therefor $W^{S(B)} \neq \emptyset$

$(g_n^m)^{-1}(M) \neq \emptyset$ then $S^{-1}(W^{S(B)} \cap (g_n^m)^{-1}(M) \neq \emptyset$ which hold $W^{S(B)} \cap (g_n^m)^{-1}(M)$is non-empty . Then S(B) is T.S of (Z, $g_{n,\infty}$)

2. Assume that B is W.M.S of (Y,$h_{n}.\infty$) , and S(x) closed subset of Y has at least two element .take r $\in N$ if $W_1^{S(B)}, W_2^{S(B)}, \dots, W_r^{S(B)}$ are non-empty open subset of S(B) and $M_1, M_2, \dots, M_r$ are non-empty open subset of Y with S(B) $\cap M_i$ is non-empty i=1,2,…,r. but S(B) is subspace of Y then$W_i^{S(B)} = W_i \cap S(B)$, for some $W_i$an open subsets of Z , for any i=1,2,…,r , but B $\cap$ S^{-1} $(W_i^{S(B)}) = B \cap S^{-1}(W_i \cap S(B)) = B \cap S^{-1}(W_i)$ (because W_i subset of S(B)) then B$\cap$ S^{-1} $(W_i^{S(B)})$are open subset of B When i =1,2,…,r. since S(B$\cap$ $S^{-1}(W_i^{S(B)})) = S(B) \cap S(S^{-1}$ $(W_i^{S(B)}) = S(B) \cap W_i^{S(B)} = W_i^{S(B)} \neq \emptyset$, (because S(B) is subspace of Y then $W_i^{S(B)}$subset of S(B)) it follows that B $\cap$ S^{-1} $(W_i^{S(B)}) \neq \emptyset$ $for\ all\ i = 1,2, \dots, r$. Furthermore $S^{-1}(W_i)$ is non-empty open subset of Y with $S^{-1}(M) \cap B \neq \emptyset$, for each i =1 ,2 ,…, r . since B is W.M.S of (Y,$h_{n,\infty}$)$\exists$n is natural numbersuch that $(S^{-1}(W_I^{S(B)}) \cap B) \cap (h_n^m)^{-1}(S^{-1}(M_i)) \neq \emptyset$, (becauseS^{-1} $(W_i^{S(B)} \cap B) \neq \emptyset$ and $S^{-1}(M) \neq \emptyset$) . As S is semi –conjugate , i.e, $g_m(S(y)) = S(h_m(y))$, For all m $\in N$ and y $\in Y$ we take invers two sided then we obtained the following $S^{-1}(g_m^{-1})(y) = (h_m^{-1})S^{-1}(y)$, for all m $\in N$, $y \in Y$ and $S^{-1}(W_i^{S(B)} \cap (g_n^m)^{-1}(M_i) \neq \emptyset$,(because$W_i^{S(B)} \neq \emptyset$ and $S^{-1}(g_n^m)^{-1}(M) \neq \emptyset$, for all i=1,2,…,m which hold $W_i^{S(B)} \cap (g_n^m)^{-1}M_i \neq \emptyset$, for all i =1 ,2 , … , m .

Then S(B) is W.M.S of (Z ,$g_{n,\infty}$)■

By above theorem , we obtained the below corollary :

Corollary 3.3 :

Assume that (Y,$h_{n,\infty}$) and (Z , $g_{n,\infty}$)be two g-NADSs . if S:Y→ Z is conjugate map , so we obtained the :

1. (Y, $h_{n,\infty}$) is T.Tif and only if (Z,$g_{n,\infty}$) has T.S .
2. (Y, $h_{n,\infty}$) is weakly mixingif and only if (Z , $g_{n,\infty}$) has W.M.S .

Definition 3.2 :

Let (Y, $h_{n,\infty}$) be g-NADS .if there exists r positive integer Number such that $h_{n+r}(y) = h_n(y)$, for any y $\in$ Y and n $\in Z^+$, then$h_{n,\infty}$ is an r-periodic discrete system.

Remark :For any r positive integernumber , let (Y, $h_{n,\infty}$) be a r-periodic discrete system . (Y , g) called induced ADS by r-periodic discrete system of (Y , $h_{n,\infty}$) if

g =$h_r \circ h_{r-1} \circ, \dots, h_n$.

Proposition 3.4 :

Assume that (Y ,$h_{n,\infty}$) $be\ a\ r - periodic$g-NADS,with (Y, g) being the matric space g = $h_r \circ h_{r-1} \circ \dots \circ h_n$

and (y , g) being the induced g- non autonomous discrete system

1. (y, $h_{n,\infty}$) has T.S , this statement always true if (y , g) has T.S .
2. (y , $h_{n,\infty}$) hasW.M.S , this statement always true if (y , g) has W.M.S.

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