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The Cubic Transmuted Survival Weibull Distribution and Application

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Abstract

In this paper new proposed distribution is shaded, cubic transmuted survival Weibull distribution, (CTSW). There for some of statistical properties are discussed and finding the estimators of its parameter. Find ingthe estimators of its parameters. Finally modeling real data setillustrated.

Keywords: Cubic transmuted weibull distribution; Cubic transmuted; Maximum likelihood; Application.

1.Introduction:

The quality of procedure utilized in statistical analysis depends heavily on assumed distribution, it has been developed large classes of standard probability distributions, however there are still many problems where the real the data not follow classical or standard probability models. Waloddi Weibull who was the first to promote the usefulness of this distribution data a sets of widely differing character (Bhardwaj et al., 2019; Murthy et al., 2004; Singh et al., 2012). Weibull distribution is a very popular life time probability distribution that has been extensively utilized for modeling in the reliability, quality control, physics, medicine and others. In this paper the new formal use Weibull distribution will be applied to its flexibility modeling distribution to the data are better than standard Weibull, by the transmuted destitution (Shaw & Buckley, 2009; Tripathy & Sukla, 2018).

By compensating the survival function to produce a new, more flexible distribution.

(Shaw & Buckley, 2009) suggested a formula transmuted distribution to find new more flexible formulas. In contrast to the Gram – Charlier approach, the research included example of parametric distributions.(Aryal & Tsokos, 2011)studied generalize the two factors Weibull distribution utilizing the quadratic rank transmuted .They described the mathematical properties and what is the intended benefit of transmuted Weibull distribution it through by utilizing it to find reliability for real data.(Elbatal & Aryal, 2013) derived "transmuted additive Weibull distribution " and discussed some of characteristics properties also, it’s the hazard ratio and reliability be heavier of the transmuted additive Weibull distribution giving an example where the (TAW) is better A.W. according to their belief.

(Merovci, 2013) **s**tudied the quadratic rank transformation lindly distribution by "shawelal"and provide comprehensive description of the mathatial properties of the subject distribution . Besides behavior reliability and utility lindley transmuted distribution. For modeling reliability data was studied utilizing real data**.**(Abd El Hady, 2014)presented his research paper generalization for a two – factor distribution by utilizing the formula suggested by " Shaw" and named it **''exponential transmuted Weibull** ''(ETW) distribution. Therefore,this paper is characterized by modeling of various data. The characteristics of new model and the probability predict is utilized to predict the factors**.**(Pal & Tiensuwan, 2014)gavestudied distribution called "beta a transmuted Weibull " distribution.application of metals to real –life data have been cited and shown to given considerable good fit.(Butt et al., 2016)presentedthree factors of the transformation power function distribution are provided which is generalization power distribution. The use two set of real data to show flexibility of the new situation**.**(AL-Kadim, 2018)derived "the general formula for the transmuted distribution" is presented properties the formula helps us build models for a new distribution arrangement (FRTWD).(Mohamaad & Al-Kadim, 2021)found a new survival model is being defined new formula was applied to fields of science such as biostatistics, metrology and engineering.

three distributions The new formula may serve as a flexible transmuted to other distributions for modeling survival data arising in several

2. Transmuted distribution(AL-Kadim & Mohammed, 2017; Shaw & Buckley, 2009)

(Shaw & Buckley, 2009) have proposed quadric transmuted family of distribution with CDF

$F(t) = (1+k)G(t) - kG^2(t)$ ….. 1

Where $k\epsilon$ $[-1, 1]$ is called factor transmutation and $G(t)$is a CDF of base distribution .

The transmuted family of distribution of (1) has been recently generalized to cubic transmuted family by (AL-Kadim & Mohammed, 2017). The CDF of cubic transmuted family of distribution has the form:

$F(x) = (1+k)G(t) - 2kG^2$(t)+ k$G^3(t)|k| \leq 1$ …….. 2

$f(t) = (1+k)g(t) - 4kG(t)g(t) + 3kG^3(t)g(t)$ ……… 3

Where G(t) is the CDF of the base distribution and t $\in R$, k are transmutation factor. The cubic transmuted family is flexible enough to capture the complexity of real life data sets.

3. The New Formula of Distribution (Methodology) and the Transmuted Formula(Mohamaad & Al-Kadim, 2021)

In this suction we fined the new distribution depends on survival function (s(t)) which is defined by :

$S(t) = (T > t)$,t> 0

$$= 1 - p(T \leq t)$$

$= 1 - F(t)$……. 4

Now, on computing a new S(t) form the (2) .

The Transmuted Survival Formula:

$S(t) = (1+k)S_1^3(t) + kS_1^2(t) - 2kS_1(t)$ …… 5

Where $S_1(t)$is the survival function of base line distribution.

$k\epsilon$ $[-1, 1]$ is calledtransmuted factor .

By differentiation law

$$ds(t) = -dF(t) = -f(t)$$

We have :

$$-f(t) = -3(1+k)s_1^2(t)f(t) - 2k\, s_1(t)f(t) + 2kf(t)$$

$f(t) = 3(1+k)s_1^2(t)f_1(t) + 2k\, s_1(t)f_1(t) - 2kf_1(t)$ …….6

4. Cubic Transmuted Survival Weibull Distribution (CTSW):

$S*(t) \; = \; (1+k)\, S^3(t) + K\, S^2(t) - 2k\, S(t)$ ………. 7

The CDF of Weibull distribution:

$F(t) \; = \; 1 - \; exp^{-(\frac{t}{b})^a}$……….. 8

$f(t) = \frac{a}{b}\left(\frac{t}{b}\right)^{a-1} exp^{-(\frac{t}{b})^a}$……..9

Utilizing the pdf and cdf of (8),(9) I the respectively

Where **a** is the dimensionless shape factor ,**b** is the scale factor .

The survival of Weibull distribution

S_W(t) = 1- [1- $exp^{-(\frac{t}{b})^a}$] ………. 10

S_W(t) = $exp^{-(\frac{t}{b})^a}$ ………. 11

Now we substitute it into the formula(7) we get:

$S*(t) \; = \; (1+k)\, exp^{-3(\frac{t}{b})^a} + k\, exp^{-2(\frac{t}{b})^a} - 2k\, exp^{-(\frac{t}{b})^a}$ ………. 12

The CDF of cubic transmuted survival Weibull distribution (CTSW) is;

$F_{CTSW}(t) = 1 - S_*(t)$ ………. 13

$F_{CTSW}(t) \; = \; 1 - (1+k)\, exp^{-3(\frac{t}{b})^a} - \; k\, exp^{-2(\frac{t}{b})^a} + \; 2k\, exp^{-(\frac{t}{b})^a}$. 14

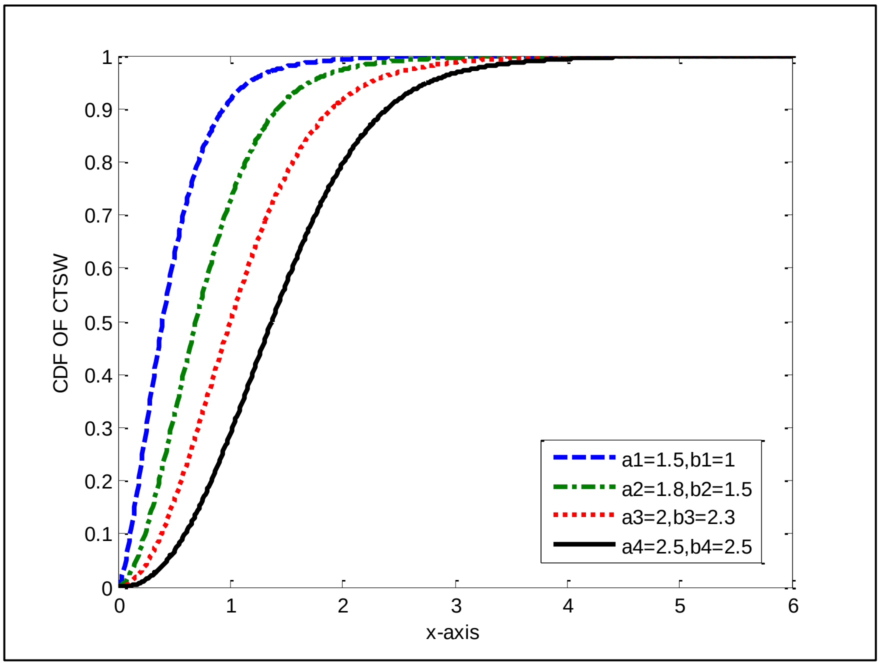


Figure (1) The CDF of the cubic transmuted survival weibull distribution with variousmagnitudes of a , b and fixed k (k=-0.06)

The pdf of (CTSW) distribution:

$$f_{ctsw}(t) = 3(k+1)\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1}\exp^{-3\left(\frac{t}{b}\right)^{a}} + 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1}\exp^{-2\left(\frac{t}{b}\right)^{a}} - 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1}\exp^{-\left(\frac{t}{b}\right)^{a}} \quad \ldots\ldots\ldots 15$$

$$= \left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1}\exp^{-\left(\frac{t}{b}\right)^{a}}\left[3(k+1)\exp^{-2\left(\frac{t}{b}\right)^{a}} + 2k\exp^{-\left(\frac{t}{b}\right)^{a}} - 2k\right] \quad \ldots\ldots 16$$

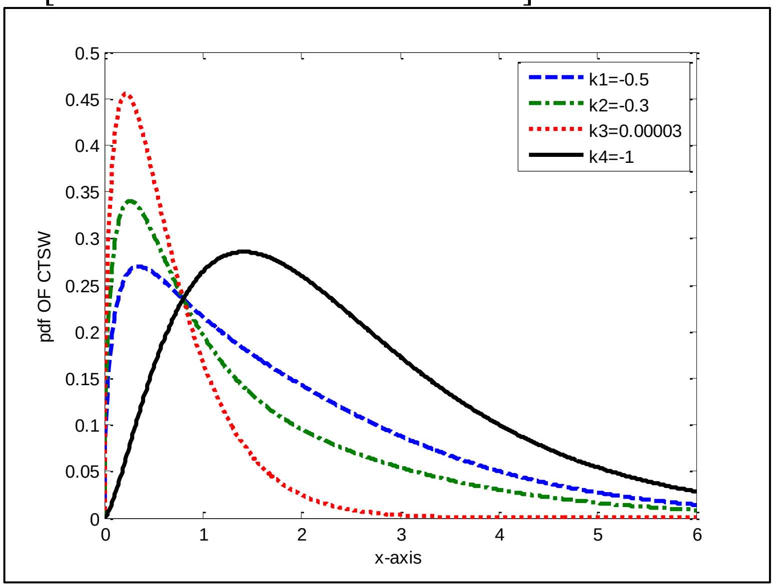


Figure (2) the pdf of the cubic transmuted survival Weibull distribution with variousmagnitudes of k , and fixed a =1.5,b=2

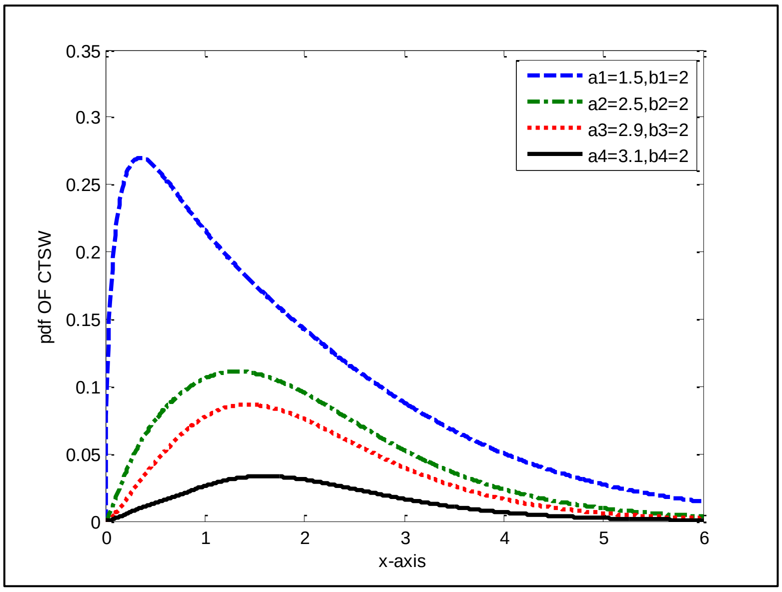


Figure (3) the pdf of the CTSW with variousmagnitudes of a, and fixed b=2, a=1.5 k=-0.5

So we can say in general that the curve of the pdf of CTSW has the same mod at fixed magnitudes of k and variousmagnitude of the factors.

Now we want to prove that the function in (15) pdf of CTSW distribution satisfied two conditions of property density function:

1. $f(t) \geq 0$.
2. $\int_{-\infty}^{\infty} f(t) = 1$.

Since the factor of TCSW a,b ≥ 0 and $k \in [-1, 1]$clear f(t) ≥ 0 .

Now

Prove 2:

$$\int_0^{\infty} f_{CTSW}(t) = \int_0^{\infty} \Big[3(k+1)\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} \exp^{-3\left(\frac{t}{b}\right)^a} + 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} \exp^{-2\left(\frac{t}{b}\right)^a} - 2k\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} \exp^{-\left(\frac{t}{b}\right)^a} \Big]\, dt.$$

$$= \left[-(k+1)\int_0^{\infty} -3\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} \exp^{-3\left(\frac{t}{b}\right)^a} - k\int_0^{\infty} -2\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} \exp^{-2\left(\frac{t}{b}\right)^a} + 2k\int_0^{\infty} \left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} \exp^{-\left(\frac{t}{b}\right)^a}\right] dt.$$

$$= [-(k+1)[\exp^{-3\left(\frac{t}{b}\right)^a}]_0^{\infty} - k[\exp^{-2\left(\frac{t}{b}\right)^a}]_0^{\infty} + 2k[\exp^{-\left(\frac{t}{b}\right)^a}]_0^{\infty}]\, dt.$$

$$= [-(k+1)[e^{-\infty} - e^0] - k[e^{-\infty} - e^0] + 2k[e^{-\infty} - e^0]]\, dt.$$

$$= (1+k)+k-2k$$

$$= 1.$$

5.The Limit of CDF and Pdf of (CTSW) Distribution:

The limit of CDF: Taking the limit for the formula (14) as

$$\lim_{t\to\infty} F_{CTSW} = \lim_{t\to\infty}\left[1-(1+k)exp^{-3\left(\frac{t}{b}\right)^a} - kexp^{-2\left(\frac{t}{b}\right)^a} + 2kexp^{-\left(\frac{t}{b}\right)^a}\right] dt \qquad \text{..... 16}$$

$$= 1$$

$$\lim_{t\to 0} F_{CTSW} = \lim_{t\to 0}\left[1-(1+k)exp^{-3\left(\frac{t}{b}\right)^a} - kexp^{-2\left(\frac{t}{b}\right)^a} + 2kexp^{-\left(\frac{t}{b}\right)^a}\right] dt \qquad \text{..... 17}$$

$$= 0$$

The limit of PDF: Now Taking the limit for the formula (15) as

$$\lim_{t\to\infty} f_{CTSW}(\mathrm{t}) = \lim_{t\to\infty}\left[3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} + 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} - 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}\right] \ldots\ldots\ 18$$

$= 0$

$$\lim_{t\to 0} f_{CTSW}(\mathrm{t}) = \lim_{t\to 0}[3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} + 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} - 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}]\ldots\ldots\ 19$$

$= 0$

This result conclusion the plot of the in figures (1.3, 1.4 ,1.5).

6 Some Function to the Survival(Rinne, 2014):

6.1 The Survival Function:

The survival function defines the probability that the element will not fail (t,0), In other word, the probability that the organism will survive until that time determinant (t), we denote by $S(t)$

The term survival function is usually utilized in medical and life studies, and its mathematical formula is:

$S(t) = p(T \geq t) = \int_t^{t\ max} p(t)\, dt \ldots\ldots$ 20.

$S(t) = 1 - p(T \geq t)$…….. 21.

$S(t) = 1 - F(t)$ ……… 22

The survival function is inversely proportional to time, in the other words:

$\lim_{t\to 0} S(t) = S(0) = 1$……23

$\lim_{t\to\infty} S(t) = S(\infty) = 0$ ……. 24

The survival function of cubic transmuted survival Weibull distribution by the formula (12):

$s(t)_{CTSW}$ =(1+k) $exp^{-3(\frac{t}{b})^a}$+k$exp^{-2(\frac{t}{b})^a}$-2k $exp^{-(\frac{t}{b})^a}$……. 25.

And

Then the $\lim_{t\to 0} S_{CTSW}(t) = S_{CTSW}(0) = 1.$

$$\lim_{t\to\infty} S_{CTSW}(t) = S_{CTSW}(\infty) = 0$$

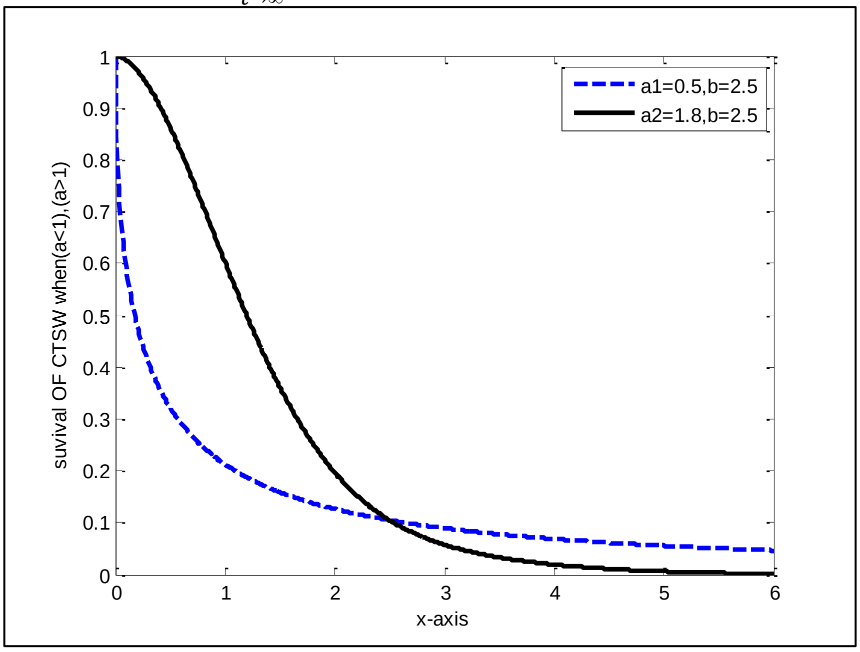


Figure (4)The survival of the cubic transmuted survival Weibull distribution (a<1 & a>1)

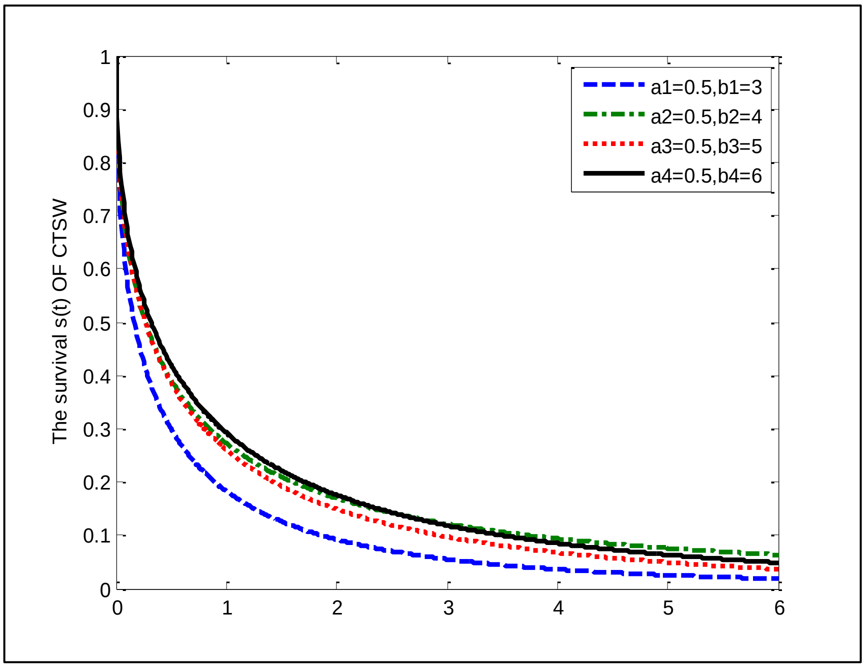


Figure (5) Thesurvival function of CTSW distribution with variousmagnitude of b,k and fixed a,(k1= - 0.01;k2= - 0.08;k3= 0.001;k4= 0.002)

6.2. The Hazard Function:

The hazard function is conditional failure rate in that it is conditional a person has actually survival until time t.

The hazard function formula is:

$h(t) = \frac{f(t)}{s(t)}$......... 26.

f(t) the probability density function of CTSW distribution.

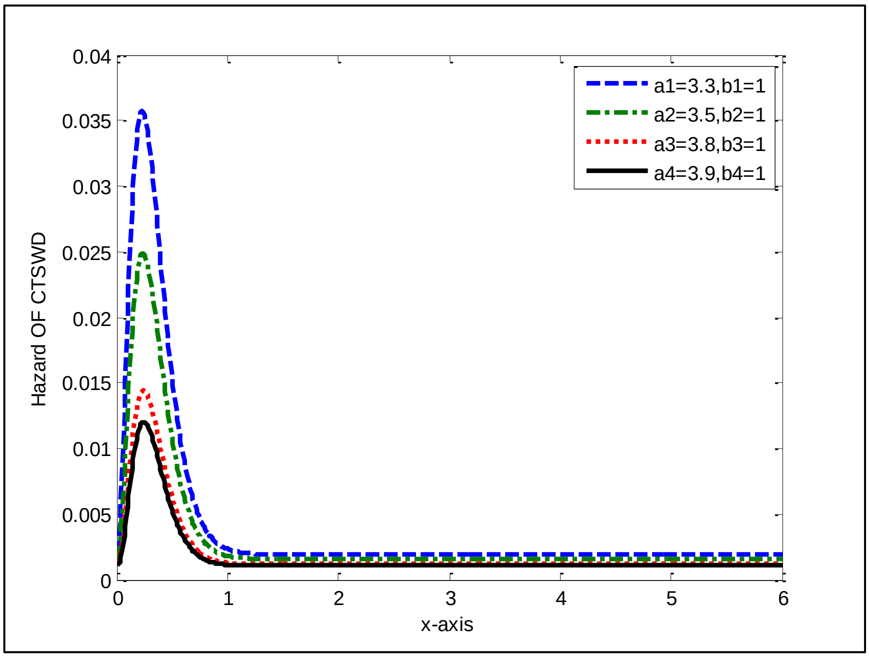
S(t) the survival function CTSW distribution.

(The probability of surviving beyond a certain point in the time t).

Now the hazard function of CTSW distribution

$$h(t) = \frac{\left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} \exp^{-\left(\frac{t}{b}\right)^{a}} \left[3(k+1)\exp^{-2\left(\frac{t}{b}\right)^{a}} + 2k\exp^{-\left(\frac{t}{b}\right)^{a}} - 2k\right]}{(\exp^{-\left(\frac{t}{b}\right)^{a}})[(1+k)\, exp^{-2\left(\frac{t}{b}\right)^{a}} + K\, exp^{-\left(\frac{t}{b}\right)^{a}} - 2k\,]} \quad \ldots\ldots\ 27.$$

$$= \left(\frac{a}{b}\right)\left(\frac{t}{b}\right)^{a-1} \frac{\left[3(k+1)\exp^{-2\left(\frac{t}{b}\right)^{a}} + 2k\exp^{-\left(\frac{t}{b}\right)^{a}} - 2k\right]}{[(1+k)\, exp^{-2\left(\frac{t}{b}\right)^{a}} + K\, exp^{-\left(\frac{t}{b}\right)^{a}} - 2k\,]} \quad \ldots\ldots..\ 28.$$



The Hazard function of CTSW distribution with variousmagnitude of a and fixed b , (k=- 0.01)

7.The Moment and quantiles(Mood, 1950):

7.1Moments of CTSW

Theorem 1: the rth moment of TSW distribution about the origin is:

$$\mathbf{E}(\boldsymbol{T}^{r}) = \boldsymbol{b}^{r}\boldsymbol{\Gamma}\left(\frac{r}{a}+\mathbf{1}\right)[\mathbf{3}^{-\frac{r}{a}}(\mathbf{1}+\boldsymbol{k})+\mathbf{2}^{-\frac{r}{a}}\boldsymbol{k}-\mathbf{2}\boldsymbol{k}]$$

Proof:

$$\mathrm{E}(T^{r}) = \int_{0}^{\infty} t^{r} f(t)dt$$

$$= \int_{0}^{\infty} t^{r}\left[3(\mathrm{k}+1)\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} + 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}} - 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}\right]dt$$

$$= \int_{0}^{\infty} 3(\mathrm{k}+1)t^{r}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-3\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}\mathrm{dt} + \int_{0}^{\infty} 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-2\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}dt$$

$$- \int_{0}^{\infty} 2\mathrm{k}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}-1}\exp^{-\left(\frac{\mathrm{t}}{\mathrm{b}}\right)^{\mathrm{a}}}]dt$$

Let $u = \left(\frac{t}{b}\right)^{a} \rightarrow u^{\frac{1}{a}} = \frac{t}{b} \rightarrow b\, u^{\frac{1}{a}} = t \rightarrow \frac{b}{a}u^{\frac{1}{a}-1}\, du = dt$

$$= \int_{0}^{\infty} 3(\mathrm{k}+1)\left(b\, u^{\frac{1}{a}}\right)^{r}\frac{a}{b}\frac{u}{u^{\frac{1}{a}}}e^{-3u}\left(\frac{b}{a}u^{\frac{1}{a}-1}\right)du + \int_{0}^{\infty} 2\mathrm{k}\left(b\, u^{\frac{1}{a}}\right)^{r}\frac{a}{b}\frac{u}{u^{\frac{1}{a}}}e^{-2u}\left(\frac{b}{a}u^{\frac{1}{a}-1}\right)du$$

$$- \int_{0}^{\infty} 2\mathrm{k}\left(b\, u^{\frac{1}{a}}\right)^{r}\frac{a}{b}\frac{u}{u^{\frac{1}{a}}}e^{-u}\left(\frac{b}{a}u^{\frac{1}{a}-1}\right)du$$

$$= \int_{0}^{\infty} 3(\mathrm{k}+1)\left(b\, u^{\frac{1}{a}}\right)^{r}e^{-3u}du + \int_{0}^{\infty} 2\mathrm{k}\left(b\, u^{\frac{1}{a}}\right)^{r}e^{-2u}du - \int_{0}^{\infty} 2\mathrm{k}\left(b\, u^{\frac{1}{a}}\right)^{r}e^{-u}du$$

$=3(\mathrm{k}+1)b^{r}\int_{0}^{\infty} u^{\frac{r}{a}}e^{-3u} + 2\mathrm{k}\, b^{r}\int_{0}^{\infty} u^{\frac{r}{a}}e^{-2u}du - 2kb^{r}\int_{0}^{\infty} u^{\frac{r}{a}}e^{-u}du$

By formula of gamma function (Sebah & Gourdon, 2002)

$$\int_{0}^{\infty} x^{\alpha-1}e^{-\lambda x}dx = \frac{\Gamma(\alpha)}{\lambda^{\alpha}}\ \lambda > 0$$

$$\int_{0}^{\infty} x^{\alpha-1}e^{-x}dx = \Gamma(\alpha)$$

$=3(\mathrm{k}+1)b^{r}\int_{0}^{\infty} u^{\left(\frac{r}{a}+1\right)-1}e^{-3u} + 2\mathrm{k}b^{r}\int_{0}^{\infty} u^{\left(\frac{r}{a}+1\right)-1}e^{-2u}du - 2kb^{r}\int_{0}^{\infty} u^{\left(\frac{r}{a}+1\right)-1}e^{-u}du.$

$=3(k+1)b^r\frac{\Gamma(\frac{r}{a}+1)}{3^{(\frac{r}{a}+1)}}+2kb^r\frac{\Gamma(\frac{r}{a}+1)}{2^{(\frac{r}{a}+1)}}-2kb^r\Gamma(\frac{r}{a}+1)$

$=b^r\Gamma\left(\frac{r}{a}+1\right)[3^{-\frac{r}{a}}(1+k)+2^{-\frac{r}{a}}k-2k]$……. 29

7.2 Mean and Variance of CTSW distribution(Mood, 1950):

by theorem (1) when (r=1) we gain mean;

$E(T)=b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]$ ………. 30

If (r=2)

$E(T^2)=b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]$ ……….31

The variance is

$var(T)=E(T^2)-[E(T)]^2$ ……….32

Now

$var(T)=b^2\Gamma\left(\frac{2}{a}+1\right)[3^{-\frac{2}{a}}(1+k)+2^{-\frac{2}{a}}k-2k]-[b\,\Gamma\left(\frac{1}{a}+1\right)[3^{-\frac{1}{a}}(1+k)+2^{-\frac{1}{a}}k-2k]\,]^2$

………. 33

8.Quantile Function of CTSW Distribution(Mohamaad & Al-Kadim, 2021).

Consider the cdf in (14) and let:

$F_{CTSW}(t)=w$

$$1-(1+k)\,exp^{-3(\frac{t}{b})^a}-\;k\,exp^{-2(\frac{t}{b})^a}+\;2k\,exp^{-(\frac{t}{b})^a}=w$$

$$(1+k)\,exp^{-3(\frac{t}{b})^a}+\;K\,exp^{-2(\frac{t}{b})^a}-\;2k\,exp^{-(\frac{t}{b})^a}=1-w$$

By utilizing Taylor series representation, we get:

$$(1+k)\sum_{n=0}^{\infty}\frac{(-3\left(\frac{t}{b}\right)^a)^n}{n!}+k\sum_{n=0}^{\infty}\frac{(-2\left(\frac{t}{b}\right)^a)^n}{n!}-2k\sum_{n=0}^{\infty}\frac{(-\left(\frac{t}{b}\right)^a)^n}{n!}=1-w$$

$$(1+k)\sum_{n=0}^{\infty}\frac{(-3)^n(\frac{t}{b})^{an}}{n!}+k\sum_{n=0}^{\infty}\frac{(-2)^n(\frac{t}{b})^{an}}{n!}-2k\sum_{n=0}^{\infty}\frac{(-1)^n(\frac{t}{b})^{an}}{n!}=1-w$$

$$(1+k)\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-3)^n}{n!}+k\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-2)^n}{n!}-2k\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-1)^n}{n!}=1-w$$

$$\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-3)^n}{n!}+k\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-3)^n}{n!}+k\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-2)^n}{n!}-2k\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-1)^n}{n!}=1-w$$

$\sum_{n=0}^{\infty}\left(\frac{t}{b}\right)^{an}\frac{(-3)^n+k((-3)^n+(-2)^n-2(-1)^n)}{n!}=1-w$ ….. 34

Finally, the formula (34) can solve numerically for t.

9.Maximum Likelihood Predicts (Hogg et al., 1977)

Let t1,t2,…..,tn be random sample of size n .ϑ is noted as the cubic transmuted survival Weibull distribution factors . which are be predicted namely $\vartheta=(a,b,k)$ sample is given by p.d.f CTSW is

$$(\frac{a}{b})(\frac{t}{b})^{a-1}e^{-(\frac{t}{b})^a}[\,3(1+k)e^{-2(\frac{t}{b})^a}+2ke^{-(\frac{t}{b})^a}-2k]$$

$$(\frac{a}{b^a})t^{a-1}e^{-(\frac{t}{b})^a}[\,3(1+k)e^{-2(\frac{t}{b})^a}+2ke^{-(\frac{t}{b})^a}-2k]$$

$L=(\frac{a}{b^a})^n\exp\left(\frac{1}{b^a}\sum_{i=1}^{\infty}t_i^a\right)\prod_{i=1}^{n}t_i^{a-1}[\,3(1+k)e^{-2(\frac{t_i}{b})^a}+2k\,e^{-(\frac{t_i}{b})^a}-2k]$….. 35

And the ln- likehood function $\ln L=lnL\,(t_1,t_2,\cdots,t_n:a,b,k)$of this random sample is given by

$$lnL=nln\frac{a}{b^a}+\frac{1}{b^a}\sum_{i=1}^{n}t_i^a+(a-1)\sum_{i=1}^{n}lnt_i+\sum_{i=1}^{n}\ln[3(1+k)e^{-2\left(\frac{t_i}{b}\right)^a}+2ke^{-\left(\frac{t_i}{b}\right)^a}-2k]$$

……36

Differentiating above formula with respect to factor a ,b ,and k respectively and equating each derivative to zero we gain the formulas

$$n\left(\frac{1}{a}-\frac{alnb}{b-a}\right)+\sum_{i=1}^{n}[t_i^a \ln(t_i)\, b^{-a}-t_i^a\, b^{-a} lnb]+\sum_{i=1}^{n}\ln t_i +$$
$$\sum_{i=1}^{n}\frac{3(1+k)(-2t_i^a \ln(t_i)b^{-a}+2t_i^a\; b^{-a}\ln(b)\exp(-2t_i^a\, b^{-a})}{3(1+k)e^{-2\left(\frac{t_i}{b}\right)^a}+2ke^{-\left(\frac{t_i}{b}\right)^a}-2k}+\frac{2k-t_i^a\, ln(t_i)b^{-a}+t_i^a\; b^{-a}\ln(b)\exp(-t_i^a\, b^{-a})}{3(1+k)e^{-2\left(\frac{t_i}{b}\right)^a}+2ke^{-\left(\frac{t_i}{b}\right)^a}-2k}=0\text{..... } 37$$
$$-\frac{a}{b}+\frac{a\sum_{i=1}^{n}t_i}{b^{a+1}}+\sum_{i=1}^{n}\frac{3(k+1)(2at^a e^{-2t^a}b^{-a}b^{-(a+1)}+2k(at^a b^{-(a+1)}e^{-(t^a b^{-a})})}{3(1+k)e^{-2\left(\frac{t_i}{b}\right)^a}+2ke^{-\left(\frac{t_i}{b}\right)^a}-2k}=0\text{...... } 38$$
$$\sum_{i=1}^{n}\frac{3exp^{-2\left(\frac{t_i}{b}\right)^a}+2exp^{-\left(\frac{t_i}{b}\right)^a}-2}{3(1+k)e^{-2\left(\frac{t_i}{b}\right)^a}+2ke^{-\left(\frac{t_i}{b}\right)^a}-2k}=0\text{.......... } 39$$

The MLM predicts $(\hat{a}, \hat{b}, \hat{k})$factors a,b,k are gainedinteravely from the nonlinear system of formulas (1,2,3).

Application

In this section, we show how the CTSW was utilized to analyze real data sets and compare it to the TWD(Pobočíková et al., 2018).

To compare the distributions, we use the following criteria:

- calculating the Akaike's information criterion (AIC) (Akaike, 1974)
  $AIC=-2Ln(\widehat{\vartheta})+2m$ ….. 40
  Where $L(\widehat{\vartheta})=L(t_1, t_2, \cdots, : \hat{\vartheta})$ For the predicted model, is the maximum magnitude of the likelihood function $\hat{\vartheta}$ is MLM predict of the factorϑ , m is number of factors to be predicted, n is number of observed data
- calculating the updated Akaike's information criterion (AICC) (Hurvich & Tsai, 1989).
  $AICC=AIC+\frac{2m(m+1)}{n-m+1}$ ……….. 41
- the Bayesian information criterion (BIC) – determinedutilizing(Schwarz, 1978).
  $BIC=-2\,Ln(\widehat{\vartheta})+m\,Ln(n)$ ………..42
- the root mean square error (RMSE) – determinedutilizing:
  $RMSE=\left[\frac{1}{n}\sum_{i=1}^{n}(F_n(x_i)-F(x_i))^2\right]^{1/2}$ ………..43

Smaller AIC, BIC, AICC, and RMSE magnitudes correlate to a distribution that fits the data better.

Data set 1

The first data set represents lifetimes of Kevlar 49/epoxy strands subjected to constant sustained pressure at 90% stress level until the strand failure The data are as follows:
"0.0251, 0.0886, 0.0891, 0.2501, 0.3113, 0.3451, 0.4763, 0.5650, 0.5671, 0.6566, 0.6748, 0.6751, 0.6753, 0.7696, 0.8375, 0.8391, 0.8425, 0.8645, 0.8851, 0.9113, 0.9120, 0.9836, 1.0483, 1.0596, 1.0773, 1.1733, 1.2570, 1.2766, 1.2985, 1.3211, 1.3503, 1.3551, 1.4595, 1.4880, 1.5728, 1.5733, 1.7083, 1.7263, 1.7460, 1.7630, 1.7746, 1.8275, 1.8375, 1.8503, 1.8808, 1.8878, 1.8881, 1.9316, 1.9558, 2.0048, 2.0408, 2.0903, 2.1093, 2.1330, 2.2100, 2.2460, 2.2878, 2.3203, 2.3470, 2.3513, 2.4951, 2.5260, 2.9911, 3.0256, 3.2678, 3.4045, 3.4846, 3.7433, 3.7455, 3.9143, 4.8073, 5.4005, 5.4435, 5.5295, 6.5541, 9.0960."

Table 1. MLM predicts of the factors and model selection criteria for data set 1

Distribution	Factorpre dicts	Statistical test				
		ln *L*	***AIC***	***AICC***	***BIC***	***RMSE***
$w(a, b)$	$\hat{a}$=1.3256 $\hat{b}$=2.1328	-122.5247	249.0494	249.2094	253.7108	0.0423
$w(a, b, c)$	$\hat{a}$=1.3169 $\hat{b}$=2.1228 $\hat{c}$=0.0058	-122.5141	251.0282	251.3525	258.0204	0.0425
$w(a, b, \lambda)$	$\hat{a}$=1.0509 $\hat{b}$=1.4419 $\hat{\lambda}$=-0.7955	-121.4300	248.8600	249.1843	255.8522	0.0351
$w(a, b, k)$	$\hat{a}$=0.1490	-120.1507	246.3015	246.6258	253.2937	0.0228

	$\hat{b}$=41.2770 $\hat{k}$=-0.8310					

In Table 1 we see that magnitudes of AIC, AICC and RMSE are smallest ones and magnitude of ln L for CTSW distribution, compared to magnitudes for 2-factor and 3- factor WDs and CTWD. Hence, we can conclude that the CTSW distribution provides better fit to the data than the other three distributions.
Data set 2
The second data set represents survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli are as follows:
"10, 33, 44, 56, 59, 72, 74, 77, 92, 93, 96, 100, 100, 102, 105, 107, 107, 108, 108, 108, 109, 112, 113, 115, 116, 120, 121, 122, 122, 124, 130, 134, 136, 139, 144, 146, 153, 159, 160, 163, 163, 168, 171, 172, 176, 183, 195, 196, 197, 202, 213, 215, 216, 222, 230, 231, 240, 245, 251, 253, 254, 254, 278, 293, 327, 342, 347, 361, 402, 432, 458, 555."

Table 2. MLM predicts of the factors and model selection criteria for data set 2

Distribution	**Factorpre dicts**	**Statistical test**				
		ln *L*	***AIC***	***AICC***	***BIC***	***RMSE***
$w(a, b)$	$\hat{a}$= 1.8253 $\hat{b}$= 199.6177	-427.3700	858.7400	858.9090	863.2933	0.0477
$w(a, b, c)$	$\hat{a}$= 1.7610 $\hat{b}$= 193.3730 $\hat{c}$= 5.0001	-427.1846	860.3692	860.7121	867.1992	0.0463
$w(a, b, \lambda)$	$\hat{a}$= 1.3534 $\hat{b}$= 139.6319 $\hat{\lambda}$= -0.9506	-425.6935	857.3870	857.7299	864.2170	0.0382
$w(a, b, k)$	$\hat{a}$=1.2718 $\hat{b}$=405.863 $\hat{k}$=-0.0696	-335.9062	677.8124	678.1553	684.642	5.3018e-07

criteria as the Akaike's information criterion, the corrected Akaike's information criterion, the Bayesian information criterion, the coefficient of determination and the root mean square error. We have demonstrated that the cubic transmuted survival weibull distribution is more flexible than the other distributions we compared it against.

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In Table 2 we see that magnitudes of AIC, AICC and RMSE are smallest ones and magnitude of ln L for CTSW distribution, compared to magnitudes for 2-factor and 3-factor WDs and CTWD. Hence, we can conclude that the CTSW distribution provides better fit to the data than the other three distributions.

Conclusion

In this paper, new distribution is derived for the transmuted Weibull distribution called cubic transmuted survival Weibull distribution. Some of its mathematical and statistical properties are discussed. The usefulness of this distribution for modelling lifetime was illustrated utilizing two real data sets and compared its performance of with the transmuted Weibull distribution, the 2-factor and the 3-factor Weibull distributions. Comparison of distributions was based on

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