Students' Metacognitive Levels In Mathematical Problem Solving: The Study Of Initial Mathematical Competence And Gender

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Abstract

This research work sought to seek and describe students' metacognitive levels in mathematical problem solving discussed from their initial mathematical competence and gender. This qualitative study applied phenomenon research design which reported the phenomenon found systematically, rigorously, and deeply. The participants involved as the research subjects were three male and three female students of Mathematics major, Faculty of Science, Universitas Muhammadiyah Jakarta, Indonesia, in the academic year of 2020/2021. These students were in initial mathematical competence of high, medium, and low levels. In collecting the data, three instruments were used; test, observation, and interview. The findings of the research showed that male and female students with high initial mathematical competence were able to solve the problem well, and able to do comprehensive evaluation towards their works so that they were classified into Reflective Use category. The male participant with medium initial mathematical competence was aware of mistake but not able yet to decide and correct that mistake, hence he was classified into Semi-strategic Use category. The female participant with medium initial mathematical competence did rechecking only after the final result was obtained and made mistake when making the conclusion, hence she was classified into Semi-reflective Use category. The male participant with low initial mathematical competence was only aware of his weakness without knowing the solution, thus, he was classified into Aware Use category. The female participant with low initial mathematical competence did not know her weakness, did not even know what she did not know, hence she was classified into Tacit Use category. These findings revealed that students' metacognitive levels were different based on gender and initial mathematical competence except those whose initial mathematical competence were high; both male and female students had the same metacognitive level.

Keywords: - metacognitive levels, problem solving, initial mathematical competence, gender difference

INTRODUCTION

Learning mathematics is a thinking process in compiling obtained information, storing and representing them. These processes are called metacognitive activities. Cognitive is a verb which shows a thinking process (Chairani, 2016). In a process of thinking, there are ways of human think. The understanding of those thinking ways called as metacognitive.

The role of metacognitive is as problem solving process expected to be mastered by university students. The concept of metacognitive itself, basically, is digging someone's thought about thinking or "thinking about thinking". As Mageira & Zawojewski (2011) opined that in the context of solving problems, metacognitive is classified into

metacognitive awareness, evaluating, regulating. Metacognitive awareness happens when the students realize to think of their knowledge position when facing a problem, together with the strategy that can be used to overcome that problem. When the students are aware to consider the limitations of their knowledge, strategy taken, and the quality of the results, they are in the stage of metacognitive evaluation. Meanwhile, when the students rethink what they are thinking in order to design plan, Determine the objectives and the steps to be implemented, then they are in the position of metacognitive regulation.

Research results showed that the success of someone in solving mathematical problem was influenced by his/her metacognitive activities

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(Yong & Kiong, 2006). This is interesting because the ability to solve mathematical problems is expected by the students after they learn mathematics, however, in Indonesia, the knowledge of metacognitive concept in solving mathematical problems is not widely discussed yet.

By using metacognitive in the process of solving problems, the students will be able to know what they have to cope with, help them see the real problems, and also understand the ways to find the solutions (Kuzle, 2013). In general, metacognitive is a key to successfully solve problems (Siegel, 2012). Therefore, it is important to be discussed so that it will become the focus of attention and open students' insights about the importance of mastering metacognitive process in dealing with mathematical problems.

There are two important metacognitive skills in solving mathematical problems, namely: selfmonitoring and planning. Self-monitoring refers to individual's ability to directly check the process of problem solving. Meanwhile, planning involves solving complex problems into several subobjectives so that they can be solved separately and sequentially until final solution. In solving mathematical problems, metacognitive helps the students deal with those problems to show them that there is a problem needs to overcome, differentiate what are the real problems and understand how to reach the objective or what solution to take (Kuzle, 2013). Metacognitive offers the possibilities for the students to adjust their actions along the process of solving mathematical problems.

Problem solving is a matter which requires the processes of high level mental and complex thinking to overcome problem. This is in line with what Gagne (1980) claimed that problem solving is a stage of thinking which takes the highest level among eight (8) types of learning. Those eight learning styles are learning the signal, response stimulus, sequence, verbal association, discrimination, concept, rules, and problem solving. As Gagne suggested, the main point of education is that teaching people to think, use their rational power, to solve problems better.

Metacognitive has important role to support the success of students in overcoming mathematical problems. This is in line with what Jianto (2020) explained that metacognitive competence is someone's ability to review, observe, and monitor the solution process of problem solving. This ability is closely related to the competence of

solving the problems. Both are interrelated, and one of the examples in solving problems, for instance, in the stage of understanding problems, metacognitive process happens on how someone understands the problem, why he/she chooses that way, how he/she identifies the data when understanding that problem and why he/she does not take other way except the one that he/she has planned. This shows that in solving problems, metacognitive competence is needed.

Referring to the study of Khairunnisa & Setvaningsih (2017), it was found metacognitive competence was not used properly by male-students because it only fulfilled the planning stage. Meanwhile, female students had used their metacognitive well in solving problems because it fulfilled three stages of metacognitive competence. These research findings were accordance with the study conducted by Anggraeni & Herdiman (2018) which found that the ability of female students in solving mathematical problems was better than the male ones because those female students were better in managing their time compared to their male-mates. Conversely, a research work done by Weni (2019) had different conclusion from the two research studies aforementioned above in which there was no difference found in terms of metacognitive competence between male and female students, they had their own characteristics to find appropriate strategies to cope with problems.

Based on different findings from those prior research studies, this research explored how university students' metacognitive competence in solving mathematical questions of problem solving in terms of initial ability and gender difference. It is hoped that the results of this study will become new sources of information and knowledge for the readers. This research study also analyzed the process of students' metacognitive competence in solving mathematical problems by measuring the relationship between those two interrelated aspects as the implementation of theories into practice. Students' levels of metacognitive were discussed based on their process in accomplishing the questions of problem solving.

METHODS

This research applied qualitative method with phenomenology design. Qualitative research is based on the philosophy of post-positivism, used to scrutinize natural condition of the object (s), where the researcher as key instrument, data is collected by triangulation technique, data analysis is qualitative, and this qualitative research results

focus more on meaning than generalization (Creswell, 2015). The participants of this research study were all 6th semester pre-service students of academic year 2020/2021, Mathematics major, Faculty of Education, Universitas Muhammadiyah Jakarta, Indonesia.

Phenomenology design used was hermeneutic. This type of qualitative design was firstly developed by Paul Ricoeur (1991) who explained that this approach aimed to understand a phenomenon systematically, rigorously, and deeply, and does not only on the surface part. This approach was selected to be applied in this research work because it was considered important to integrate the analysis of experience and meaning, and also the meaning related to that experience. These two views would complete one and another. Phenomenology cannot understand phenomenon in a complete and thorough aspect without giving the meaning to the experiences of the participants.

The selection of the research subjects was based on the criteria that these students had studied Algebra Linear and Basic Mathematics in their previous semesters as those two subjects became the pre-Program. requisite courses for Linear Comprehensive analysis was conducted by measuring initial mathematical competence of the students which consisted of three levels, i.e., high, medium, and low. These categories were taken from students' achievements in the two prerequisite subjects as mentioned above, namely Basic Mathematics and Algebra Linear. The criteria presented in Table 1.1 below referred to criteria analysis of Efendi (2016):

Table 1.1 Initial Mathematical Competence (IMC)

Interval Score Test of IMC	Category
Score MIC $\geq \bar{x} + 0.5 \text{ S}$	High
\bar{x} - 0,5 S \leq Score MIC $< \bar{x}$ + 0.5 S	Medium
Score MIC $< \overline{x} - 0.5 \text{ S}$	Low

Further this IMC was differentiated between male students and female ones with the following remarks:

 \bar{x} = Mean score (of pre-requisite subjects)

S = Deviant

Based on the criteria described above, there were six students chosen, hence this research subjects consisting of three male and three female students in which each of them had initial mathematical competence of high, medium, and low. Data were collected through tests of mathematics questions about problem solving, observation, and interview. The results of the interview were analyzed through the steps of reducing the data, presenting those

data, drawing the conclusion and verifying that conclusion. The data of test questions about mathematical problem solving were analyzed based on the correct answers of each finishing indicator as stated on the answer key. The analysis referred to how corrected the participants did the questions based on the directions. The final results of the test were analyzed and descriptively presented by referring to each stage of problem solving to be classified into their metacognitive levels.

FINDINGS AND DISCUSSION

Participants of Male Students (M)

Male participant with high initial mathematical competence was coded as M1, male in medium competence was coded as M2, and the one with low competence was coded as M3. The results of data analysis of mathematical problem-solving test, observation, and interview shown in each step involved in the cognitive process of problem solving.

Planning

At the planning stage, M1 was able to fulfill all indicators; reading and understanding problems, identifying known and questioned cases, stating those known and asked information into mathematical sentences, and thinking of possible alternative strategies to solve the problems. These were supported by the theories of scholars (Livingston, 2003; Flavel, 1979; Garofalo & Lester, 1985; Lee & Baylor, 2006) and adapted by Cohors-Fresenborg and Kaune (2007) who argued that planning is related to intentional activity which organizes all process of learning. The same thing was also stated by OLRC News (2004) that the ability to plan learning activity followed by arranging strategies to analyze the information related to learning process is very important to do in the planning process.

Based on the description above, it was understood that M1 had awareness towards the knowledge he had and able to relate it with the questions so that M1 was competent to write and explain the steps taken to overcome the cases. That awareness was metacognitive activity in planning the problem solving. This was related to the research of Sudia (2014) who found that male subjects had involved their metacognitive since they had known how to design path plan of solving the problems, formulated the formula and time-plan, and also found ways to overcome the problems.

At the planning stage, M2 was able to identify anything he had known and was asked by mentioning the information completely, however, he was still unable to state the information correctly into mathematical sentences, did not explain what was being questioned, and also unable to think of alternative solutions for the problems. If it was referred to the theories of several prior studies (Livingston, 2003; Flavel, 1979; Garofalo & Lester, 1985; Lee & Baylor, 2006), adapted by Cohors-Fresenborg and Kaune (2007), which explained that planning is related to intentional activity that organizes all process of learning, hence, M2 had not organized part of the learning process completely so that it would affect the next stage.

At the planning stage, M3 was not able yet to identify what he had known and was asked because he did not mention the information completely, was not able yet to tell the information completely into mathematical sentences, and besides, he was unable to think of any solution. If it was referred to the theories of several prior studies (Livingston, 2003; Flavel, 1979; Garofalo & Lester, 1985; Lee & Baylor, 2006), adapted by Cohors-Fresenborg and Kaune (2007), which explained that planning is related to intentional activity that organizes all process of learning, hence, M3 had not organized all of the learning process well so that it would affect the next stage.

Monitoring

At monitoring stage, M1 was able to fulfill all indicators, i.e., choosing appropriate strategy, applying that strategy in overcoming problems and finding solutions for those problems accurately and carefully. As theories (Lee and Baylor, 2006; Cohors-Fresenborg and Kaune, 2007) explained that monitoring stage is related to the activities which direct the series of learning progress. Thus, it was said that M1 had achieved metacognitive indicator at monitoring stage in which he was able to answer related questions with the steps to cope with the problems. In line with this, according to Safitri, et.al., (2020), the students involve their metacognitive in accomplishing their plan, and they will be able to monitor each step taken in solving the problems.

At monitoring stage, M2 was not able yet to fulfill the indicators of choosing appropriate strategy and using that strategy to solve problems, also, he had not fulfilled the indicator of applying strategy as the solution to deal with problems accurately and carefully because this participant did not draw the graphic in thorough so that drawing process was not finished yet and stopped. Looking back to the theories (Lee and Baylor, 2006; Cohors-Fresenborg and Kaune, 2007), it was said that monitoring stage is related to the activities which direct the series of learning progress, thus, M2's learning progress was considered as not well directed yet.

At monitoring stage, M3 was only able to fulfill the indicators of choosing appropriate strategy and applying that strategy to solve problems, however, he was unable to fulfill the indicator of using strategy to find solutions to solve the problem accurately and carefully because the participant did not draw the graphic in thorough so that that drawing process was not completed yet. Referring to the theories (Lee and Baylor, 2006; Cohors-Fresenborg and Kaune, 2007) which described that monitoring stage is related to activities which direct the series of learning progress, it could be said that M3's learning progress was not welldirected yet in which his graphic picture was not complete yet and he did not shade the part that satisfied, also, M3 had not used strategy to find solution to solve problems accurately and in thorough. The following figure 1 is part of M3's answer:

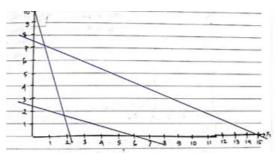


Figure 1. Part of M3's Answer

This was also in line with the opinions of Khairunnisa & Setyaningsih (2017) who stated that when students are able to understand the problems well, it means that these students have involved their metacognitive well in understanding problems. M3 had not used strategy yet to get solution to solve problems accurately and in thorough as it is shown in Figure 2 below:

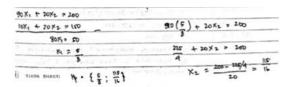


Figure 2. Part of M3's Answer

It can be seen clearly from part of his answer above that M3 did not pass all process at monitoring stage in which he was unable to choose appropriate strategy to solve problems as it was found in his

incorrect answer. Further, M3's interview results at monitoring stage can be seen in the following Table 1.2:

Table 1.2 The Excerpts of M3's Interview

Transcription

Stage	Code	Question	Answer
Monitoring	M1	2.1 How did you choose the appropriate strategy?	I selected appropriate strategy that I remembered, it was by finding out cut-off point but I forgot to determine the shading area.
	M2	2.2 How did you apply a strategy to solve problems?	I followed the steps to get the cut-off point to solve problem but I passed the step in which I had to determine the finishing area.
	МЗ	2.3 Did you use strategy to find solutions to deal with given problems accurately and carefully?	Yes, during the process of solving the problems, I went back seeing what was being questioned to confirm whether or not the solution I chose was appropriate.

From his responses to the questions in the interview, M3 was doubtful when giving his answers. This was also proven from his test's results and interview in which he consistently gave answers even they were not right. As well as from the observation results in which he was confused, seen from his gaze and gestures.

Evaluation and Confirmation

At evaluation stage, M1 was able to fulfill all indicators, namely re-checking the process of solving the problems, confirming the solution got from the process of solving those problems, deducing the relevance of the solution towards the problems given and evaluating the mathematical problems solved. M1 had achieved all the indicators of metacognitive in evaluation stage by explaining his belief about the results of problem solving, explaining the plan to do the rechecking and explaining whether or not there was a way to do that recheck. Because M1 was able to reach all indicators of metacognitive in evaluation stage, it was said that he had involved his metacognitive in evaluation. The following Figure 3 was part of M1's answer:

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10x, + 20x = 150	90 (5/0) 420 x 2:200
Jox, : 50	10 + 20x2; 200
7. > TO/80	4 20× + : 200-225
×1=5/0	2041:575 - 115
	4 16
He. 55/0, "5/167	
	1 + 360×2 =D 2 msn
	0) + 560 (10) : 3. 600
(5/0 / "5/16) M20(510) +360(115/11)=2.850 J
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Figure 3. Part of M1's Answer

At this evaluation stage, it was clearly seen that M1 was able to answer correctly and give explanation or argument precisely at each step of finishing strategy. M1 was able to evaluate the ways of

solving mathematical problems he had done and made conclusion correctly. These things were also suitable with his responses in the interview as presented in the transcription excerpts in Table 1.3 below:

 Table
 1.3
 M1's
 Excerpts
 of
 Interview

Transcription

Transcription										
Stage	Code	Question	Answer							
Evaluation	E1	3.1 Did you recheck the	Yes, I did and the answers were correct							
and		accomplished problems	already.							
Confirmation		that you have passed?								
	E2	3.2 How did you confirm	From the solution that I got, I went back to							
		the solution got from	the questions asked checking if my							
		finishing process?	answers were already right or not, and							
			when I thought it was correct, then I did							
			not recheck the finishing process again.							
	E3	3.3 How did you conclude the relevance of the solution towards the given problems?	The solution that I got was that the breeder should produce and buy com and bean sprouts in the amount of 5/8kg and 115/16kg for each, and these were relevant with the questions asked how much com and bean sprouts that have to be provide by the breeder with the minimum costs? And from several critical points I found that the minimum cost should be spent by the breeder was \$2850.							
	E4	3.4 Did you evaluate mathematical problem solving that you have taken?	Yes, in which the numbers of corn and bean sprouts that had to be produced were 5/8 and 115/16 and the cost spent by the breeder was \$2850.							

When having the interview, M1 answered all questions correctly which showed that both the test and interview's results were consistent. His correct answers were also indicated in the evaluation step in which he said that the minimum cost should be spent was \$2850. He was also able to conclude the relevance of the solution towards the given problems and evaluate mathematical problem solving that he had done.

At the evaluation stage, M2 had not rechecked the process of problem solving that he had taken in doing the questions. He was too hurry and forgot to draw the graphic, but, in fact, he was also able to confirm the relevance of the solution towards the given problems and fulfilled the indicator of evaluating mathematical problem solving that he had done.

At the evaluation stage, M3 had not fulfilled the indicator of rechecking the process of problem solving that he had done because he was in a hurry and forgot to draw the graphic, he had not confirmed the solution found yet with finishing process since he misunderstood the questions, besides, he had not fulfilled the indicator which concluded the relevance of the solution towards the given problems because the finishing process was less precise and did not meet the indicator of evaluating mathematical problem solving that he had done.

Synthesis Analysis

If referring to the theories of some experts (Livingston, 2003; Flavel, 1979; Garofalo & Lester, 1985; Lee & Baylor, 2006) adapted by

Cohors-Fresenborg and Kaune (2007), who claimed that planning is related to intentional activities which organize all process of learning, this means that M2 and M3 had not organized all learning process well so that it would affect the next stage. At monitoring stage, these two participants were only able to fulfill the indicators of selecting strategy and then applying that strategy in overcoming problems, but had not fulfilled the indicators of using strategy to get the solution to solve the given problems accurately and carefully because they were carelessly draw the graphics so that the process was not finished yet.

Furthermore, metacognitive competence of M1 in problem solving will be discussed based of the steps in solving problems as follows: at the stage of understanding problems, M1 succeeded to know the problems well. This was indicated from his correct explanation in identifying the problems that he had written. According to Khairunnisa & Setyaningsih (2017), when the students have understood the problem well, it means that they have involved their metacognitive in dealing with problems. Based on that opinion, L1 had used his metacognitive well related to problems.

During the interview, M2 was doubt in explaining his understanding about the questions. His doubtful was clearly seen from his face while giving the explanation in which sometimes he smiled and shook his head, and even more, he also put his head down. These were reinforced by his confession who said that he was having doubts because he was afraid of making mistakes. Even though full of doubts, he kept on telling what he knew about the problems asked in the question. While for M1, he did not only explain his understanding but also the way he used for that understanding; by reading by heart repeatedly until he finally got the real problems. This was also reinforced by the results of observation in which when M1 got the question, he took longer time to jot down his identification results. He had used appropriate way to understand problems as what Sari, et.al, (2016) described that the purpose of reading questions is to get the information needed for getting the solution to finish it.

M1 was fully aware of his ways of thinking to be applied in solving the problems. This showed that M1 had achieved the indicator of metacognitive at the stage of knowing problems that is by explaining the ways to be taken to deal with problems. This is in line with the study of Sudia (2014) who found that male students were aware of the importance of thinking the way to understand problems, i.e., by reading the question repeatedly until it was clearly understood. This finding showed that male participant, at this case

M1, had involved his metacognitive in understanding problems.

Further, M1 explained that reading had been becoming his habit in understanding questions, thus, that was the reason why he firstly read before finishing the problems given in the test. This indicated that he had realized the reason to find way in solving the problems. Besides, he also said that reading was his habit, therefore he did not use another way. For him, that way was the most appropriate one. This also indicated that M1 had achieved the indicator of understanding problem that was by giving reason why he did not choose another way for dealing with the problems.

Based on the results discussed above, it can be concluded that M1 was able to explain all process of thinking in solving problems and his explanation was relevant with what he had written in answering the questions. Meanwhile, M2 had not involved his metacognitive competence in each step of solving problems process because he was not able yet to identify everything he knew and was asked, did not mention what he had known completely, did not mention what was being asked, was not able yet to state all information he had known and the questions asked in mathematical sentences, and also, he could not finish the problems. As well as M3 who had not involved all his metacognitive competence in each step of solving problems. At planning stage, M3 was not able yet to identify everything he knew and being asked because he did not completely mention the information completely, he was unable yet to tell that information in mathematical sentences correctly. At monitoring stage, M3 was only able to fulfill the indicators of choosing appropriate strategy and applying that strategy in solving the problems, however, he had not met the indicator of using strategy to the solutions to overcome the given problems both accurately and in thorough since he did not draw the graphic carefully and thus, the drawing process was not accomplished yet. Seen from their metacognitive process, it can be classified into the categories presented in Table 1.4 below:

Table 1.4 Metacognitive Levels of Male Students

Participant	Metacognitive Process										Metacognitive	
	Planning ® Monitoring (M)							Evaluation (E)			Ξ)	Level
	R1	R2	R3	R4	M1	M2	M3	E1	E2	E3	E4	
M1	✓	√	√	✓	V	V	✓	✓	√	1	✓	Reflective Use
M2	✓	√	✓			√	✓		✓	✓	✓	Semi Strategic Use
M3	✓					✓	✓		✓	✓		Aware Use

In that metacognitive levels table above, M1 was able to solve the problems well, always checked the steps and directly did the revision, and also was able to make comprehensive evaluation towards his work so that these were suitable with the

category described by Swartz and Perkins (1989) about metacognitive thinking levels that put M1 into the level of Reflective Use.

M2 was aware of his mistakes but was not able yet to decide and revise them because he was in doubt, hence his metacognitive level suitable with what Swartz and Perkins (1989) described was Semistrategic Use. Meanwhile, M3 was aware of the weakness he had, and he knew what he did not know, thus, as Swartz and Perkins (1989) described about thinking awareness levels, M3 was categorized into Aware Use level.

Participants of Female Students (F)

Female participant with low initial metacognitive competence was coded as F1, with medium level as F2, and with low level as F3. The results of data analysis from problem solving test, observation and interview revealed that F1 had involved her metacognitive competence in each step of problem solving.

Planning

At planning stage, F1 had met all the indicators of this stage, namely by reading and understanding problem, identifying the matters she had known and being asked, stating in mathematical sentences all that information and thinking of possible alternative strategy to deal with them. These were in line with the theories of Lee and Baylor (2006) who said that planning is related to intentional activities which organize all process of learning. The same thing was also stated by OLRC News (2004) that the ability of planning learning activities followed by arranging strategy to manage information regarding the carried-out learning process is very important to do at this planning process.

At the planning stage, F2 fulfilled all indicators of planning stage, i.e., reading and understating problems, identifying everything she had known and what was being asked, stating the information into mathematical sentences and thinking of any possible alternative strategies to deal with them. These were supported by the theories stated by Lee and Baylor (2006); Cohors-Fresenborg and Kaune (2007) which explained that planning is related to intentional activities which organize all process of learning. The same thing was also stated by OLRC News (2004) that planning learning activities, arranging strategies to manage all information dealt with learning process is very important to be carried out in the learning process.

At planning stage, F3 was not able yet to identify what she had known and what was being questioned because she did not mention that information completely, she was unable to state all known information into mathematical sentences correctly and did not mention the questioned case and she was unable yet to think of any possible alternative strategy to overcome the problems. If referring back to the theories of the scholars (Lee and Baylor, 2006; Cohors-Fresenborg and Kaune, 2007) which described that monitoring stage is related to the activities which lead the series of learning progress, then the progress of F3 was not well guided yet. The same thing was also stated by OLRC News (2004) which explained that the ability to plan leaning activities, arranging strategy to manage information regarding the process of carried out learning is very important in the learning process, hence it can be concluded F3 had not applied her metacognitive activities yet.

Monitoring

At monitoring stage, F1 was able to fulfill the indicators, namely selecting appropriate strategy, applying the strategy in solving the problems, and using finishing strategy to get the solution over the given problems accurately and in thorough because the finishing result was correct. F1 drew graphic and shade the finishing area to meet the requirement precisely. The result of her work is shown in figure 4 below:

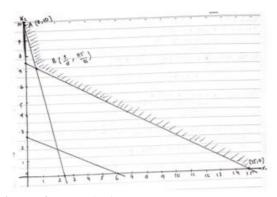


Figure 4. Part of F1's Answer

Additionally, the following Table 1.5 presented the excerpts of interview transcription at monitoring stage:

Table 1.5 F1's Excerpts of Interview Transcription

Stage	Code	Question	Answer			
Monitoring	M1	2.1 How did you choose	I chose the strategy			
		the appropriate	correctly suitable with			
		strategy?	what I have known in the			
			question and what aspect			
			that needed to find from			
			that question, and it was			
			the minimum cost.			
	M2	2.2 How did you apply	I followed the examples to			
		the strategy to solve	answer the question, the			
		problems?	steps, to coping with the			
			problem, it was by firstly			
			determining the cut-off			
			point and then I drew its			
			graphic and found its			
			critical points to see the			
			most minimum cost.			
N		2.3 Did you use finishing	Yes, during the finishing			
		strategy to get the	process of answering the			
		solution of the given	question until			
		problems accurately	accomplished, I rechecked			
		and carefully?	what was being asked and			
			made sure that I had			
			applied accurate and			
			appropriate solution.			

F1 finished her graphic drawing correctly and both her answer of the test and the results of interview at monitoring stage were consistent. As the theories proposed by scholars (Lee and Baylor, 2006; Cohors-Fresenborg and Kaune, 2007) explained that at monitoring stage all activities are directed to the series of learning progress. F1 had passed the monitoring step correctly and suitable with the indicator. Based on that, the research result of Rahmawati (2015) which stated that female students with high level of metacognitive competence at monitoring stage always do their works according to orderly plan carefully and believe that the steps are correct.

At monitoring stage, F2 was able to fulfill all indicators of selecting appropriate strategy, applying that strategy to cope with the problem and using that finishing strategy to get the solution over the given problem accurately and in thorough. These things showed that F2 had realized and understood every step of problem solving. In other words, F2 had achieved metacognitive indicator in carrying out the monitoring, i.e., able to answer the question related to the steps of problem solving. In line with this, according to Safitri, et.al., (2020), the students who involve their metacognitive ability in carrying out finishing plan will be able to monitor each step taken in solving the problems.

At monitoring stage, F3 was not able yet to meet all indicators in which she had not selected appropriate strategy, unable to apply the strategy in overcoming problem, and was not able yet to use the finishing strategy to find the solution of the given problem accurately and in thorough. F3 confessed that she was not sure in monitoring stage since she was in doubt of understanding whether or not she had carried out the previous plan correctly. It was also seen from her answers that were many mistakes as well as in drawing the graphic. F3 could not draw the graphic correctly. She even could not determine which area that she had to

shade or not, as it is shown in the following figure 5.

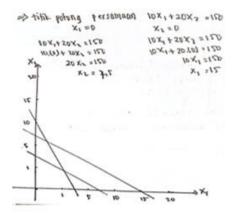


Figure 5. Part F3's Answer

Based on the description above, it can be concluded that metacognitive competence of F3 in carrying out monitoring stage to solve problems was not well-implemented yet especially in understanding her metacognitive process in interpreting the graphic picture to accomplish the questions of problem solving.

Evaluation and Confirmation

At evaluation stage, F1 was also able to fulfill all indicators, namely rechecking her way of solving the problems, confirming the solution got from the finishing process, concluding the relevance of the solution towards the given problems and evaluating the finishing process of mathematical problem that she had carried out.

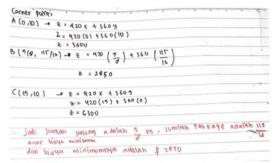


Figure 6. Part of F1's Answer

From the answers made by F1, it was seen that she had passed all process of metacognitive steps and passed all indicators of evaluation stage, i.e., rechecking the process taken in solving the problems, confirming the solution got from the finishing process and making the conclusion of the relevance of the solution towards the given problem and also evaluating the mathematical problems that she had been carried out. These were shown from her answers to the test's questions and from the excerpts of her interview interpretation as

presented in Table 1.6 below:

Table 1.6 F1's Excerpts of Interview Interpretation

erpretati							
Stage	Code	Question	Answer				
Evaluation	El	3.1 Did you recheck the	Yes, I did recheck and I				
and		finishing process of	thought my answer was				
Confirmation		problems that you	already correct.				
		have passed?					
	E2	3.2 How did you confirm	Well, from the solution				
		the solution you have	that I got, I went back to				
		got with the finishing	the questions asked in the				
		process?	test to make sure whether				
			or not it was suitable with				
			my finishing result, and				
			when I considered it was				
			suitable, I did not check				
			the finishing process again				
			since I thought it was				
			correct.				
	E3	3.3 How did you	The solution that I got was				
		conclude the	that the breeder had to				
		relevance of the	produce or buy com and				
		solution towards the	bean sprouts in the amoun				
		given problems?	of 5/8kg and 116/15 kg pe				
			each and these were				
			relevant with the questions				
			of how many kilograms				
			the breeder should provide				
			with the minimum cost?				
			and from several critical				
			points I got, the most				
			minimum cost that the				
			breeder should spend was				
			\$2850. This number was				
			evened to decimal.				
	E4	3.4 Did you evaluate the	Yes, so, each amount that				
		finishing	had to be produced for				
		mathematical	corn and bean sprouts was				
		problem that you	5/8 kg and 116/15kg with				
		have taken?	the minimum cost to spen				
			was \$2850.				

Based on the test results and interview above, it can be seen that F1's responses were consistent. She answered all problem-solving questions correctly and was able to finish them systematically according to the stages of metacognitive process. F1 made conclusion at both evaluation and confirmation stages correctly. At evaluation stage, in fully confident, F1 said that she had rechecked and recalculated her answers. When doing the evaluation step, F1 stated confidently that her finishing process was correct. The reason was that she had done the evaluation repeatedly by rechecking and recalculating the answers. When doing the evaluation, F1 expressed firmly about her belief towards her correct solution. This is in line with the research findings of Putri & Susilowati (2016), who explained that at the stage of rechecking, female students gave their beliefs to their answers firmly that the answers they did were already right. While the way F1 did her evaluation was by rechecking every step of finishing process. This indicated that she was aware of the importance of doing evaluation and understanding the way to do the evaluation.

At evaluation stage, F2 was able to fulfill the indicators, namely rechecking the finishing process of the problems taken, was aware of a little mistake she did but had not decided anything yet due the limited time, further, even F2 was also able to confirm the solution she got from the finishing process, she was not able yet to conclude the

relevance of the solution towards the given problem but she was able to evaluate the mathematical problem solving that she had carried out.

At evaluation stage, F3 had not met all the indicators since she did not recheck all process of problem solving that she had passed because she was in rush and forgot to draw the graphic, did not confirm the solution got from that finishing process towards the given problems since the finishing process was less appropriate and did not fulfill the indicator of evaluating mathematical problem solving that had been carried out and did not make any conclusion.

Synthesis Analysis

The results of data analysis of problem-solving test and interview revealed that F1 had involved her metacognitive competence in each step of problem solving. This can be seen from the interview session in which she was able to explain all her thinking process and she was able to accomplish the questions and also from the relevance of her spoken explanation with her written answers. These are in line with the research finding of Sudia (2014) who found that female students had involved their metacognitive since they were able to think the problem-solving at planning stage, thinking of the formula and the time would be used to overcome the problems and also thinking of the ways to solve the problems. These were also accordance with the opinion of Sari et al., (2016) which argued that female students' metacognitive activities with high level of mathematical competence at planning stage were related to the procedural knowledge possessed by these students towards the questions by stating the steps to be taken through looking at the given test questions.

From the description above, it was concluded that metacognitive competence of F1 in planning the problem solving was already well-used. This is in line with the research results of Fitrivah & Setianingsi (2014) who stated that female students who involved their metacognitive at planning stage would be aware of the precise steps taken or ways to get the correct problem solving. However, F2 was not accomplished the process yet as well as F3 who had not involved her metacognitive process well. These were in line with the research result of Wahyuningtyas et.al., (2019), who explained that students with high mathematical female competence were able to fulfill all their metacognitive indicators at the stages of planning, monitoring and evaluation. Seen from their metacognitive process, these female participants were classified into categories as presented in the following Table 1.5:

Table 1.5 Metacognitive Levels of Female Students

Γ	Participant	Metacognitive Process											Metacognitive
			Plan	ning		M	onitori	ng	Evaluation				Levels
		R1	R2	R3	R4	M1	M2	M3	E1	E2	E3	E4	[
Г	F1	√	~	~	√	√	~	√	√	√	✓	✓	Reflective Use
	F2	V	V	V	~	V	V	V	~	V		~	Semi Reflective Use
	F3	V											Tacit Use

From the description above, it can be concluded that F1 was able to solve the problems well, always checked every step and directly did revision, and also able to do evaluation comprehensively towards her works so that these were suitable with what explained by Swartz and Perkins (1989) about metacognitive levels, F1 could be categorized into Reflective Use level. F2 was able to finish her problems thoroughly, and then rechecked her works, was aware of her competence, did checking only after the final result was obtained and made a small mistake once making the conclusion, hence referring to thinking awareness levels of Swartz and Perkins (1989), F2 was classified into the category of Semi-reflective Use. F3 was not aware of her weakness, she had not finished the problem yet because when answering the question, she just tried, it was acknowledged that F3 did not know about what she did not know so that in the categories level of thinking awareness proposed by Swartz and Perkins (1989), F3 was classified into Tacit Use category.

With regards to what have been discussed, it can be concluded that F1 had achieved all her metacognitive indicators in solving the problems in this research study. This was in line with the research of Mayasari, et.al., (2019) who found that students with high level of mathematical competence fulfilled all metacognitive indicators of each stage of problem solving. F1 was aware of and understood her thinking process in dealing with problems. Due to that awareness and understanding, F1 was able to explain her thinking process. This also showed that F1 had involved her metacognitive process in solving the problem. This statement is based on the opinion of Chairani (2016) who said that the foundation of metacognitive competence is self-awareness towards that cognitive process.

The results of M1, M2, M3, F1, F2, and F3 in this research work become new findings because they are different from those found in prior studies. As the research results of Khairunnisa & Setyaningsih (2017) who said that metacognitive process had not used by male students well, while in this research, both male and female students with high

initial mathematical competence were able to solve the problems correctly according to all stages of their metacognitive process. This difference becomes interesting thing to discuss in further research so that new inventions will be obtained on how male and female students' cognitive competence in solving the problems when it is measured from their initial mathematical competence.

CONCLUSION

Based on the findings and discussion presented, this research concluded that both male and female participants with high level of initial mathematical competence were able to solve problems well, and able to thoroughly evaluated their work, hence they were categorized into Reflective Use. Male participant with medium level of initial mathematical competence was aware of his mistakes but still unable to decide and revise, hence he was considered into the category of Semistrategic Use. Female participant at this same metacognitive level was checking only after the final result was obtained and made mistakes in making the conclusion, hence she was classified in Semi Reflective Use category. Male participant with low level of initial mathematical competence was only aware of his weakness without knowing the solution, hence she was put in the category of Aware Use. Female participant with this same level did not know her weakness and did not even realize of what she did not understand, hence she was classified in Tacit Use category. To end, this research study reported that metacognitive levels of the students were varied based on initial mathematical competence and gender except those whose initial mathematical competence were high; either male or female, were at the same metacognitive level and category.

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